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OCTOBER 1954

School Science and Mathematics

A Journal for All Science and Mathematics Teachers

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. . . .

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Its leaves grew green and fair,
And waved in hot midsummer's noon
Its soft and yellow hair.

And now, with Autumn's moonlit eves,
Its harvest time has come,
We pluck away the frosted leaves,
And bear the treasure home.

—*"The Corn Song"*
JOHN GREENLEAF WHITTIER

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School Science and Mathematics

We Cover the Earth

Is read by subscribers in every state of the Union, all provinces of Canada, and thirty-three foreign countries.
It is owned by teachers, managed by teachers, and edited by teachers.
It comes to your desk every school month.

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SCHOOL SCIENCE
AND
MATHEMATICS

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BERMUDA—FROM TEMPEST ISLE TO TOURIST HAVEN

MINNIE E. LAMAIRE
Mount Holyoke College, South Hadley, Mass.

Miranda. If by your art, my dearest father, you have
Put the wild waters in this roar, allay them.
The sky, it seems, would pour down stinking pitch,
But that the sea, mounting to the welkin's cheek,
Dashes the fire out. O, I have suffer'd
With those that I saw suffer! A brave vessel,
Who had, no doubt, some noble creatures in her,
Dash'd all to pieces.
Act I. Scene 2 of "The Tempest" by William Shakespeare.

Bermuda takes its name from Juan de Bermudez, a Spaniard who had discovered the islands by 1511 deduced from the fact that at this time a map dated in that year appeared indicating the name of the group as "La Bermuda." The Spaniards showed no interest in the islands and did not attempt colonization. With the settlement of the Virginia colony the islands took on more importance for they were along the route to the new continent. Admiral Sir George Somers and a party of Virginia colonists, encountering a very bad storm, were wrecked on the islands in 1609. It was the story and description of this wreck taken from an account by William Strachey, secretary-elect for Virginia, that gave Shakespeare his background and inspiration for the writing of "The Tempest." Bermuda was first colonized by the Virginia Company and later by the Bermuda Company, with the first colonists reaching the islands on July 12, 1612.

The Bermuda Islands (Figure 1), consisting of some 150 islands and islets, represent the most northerly group of coral islands in the world today. The foundation of this grouping derives from a crest of a vast submarine volcanic mountain rising some 15,000′ from the

Atlantic depths with the top 245′ below sea level at present. Limestone developed on top of this base and, in turn, was overlaid with a thick coral growth. The limestone and coral are not, in general, found above sea level but are visible and available for study in the many caves that permeate the islands, one of which is Prospero's Cave, reputed scene of "The Tempest." Nearly all the rocks above sea level are formed of consolidated sand dunes made up of wind-drifted shell sand. In the open cuts the irregular stratification of the beds, many times at angles approaching ninety degrees, clearly demonstrates the characteristic features of eolian sand dunes.

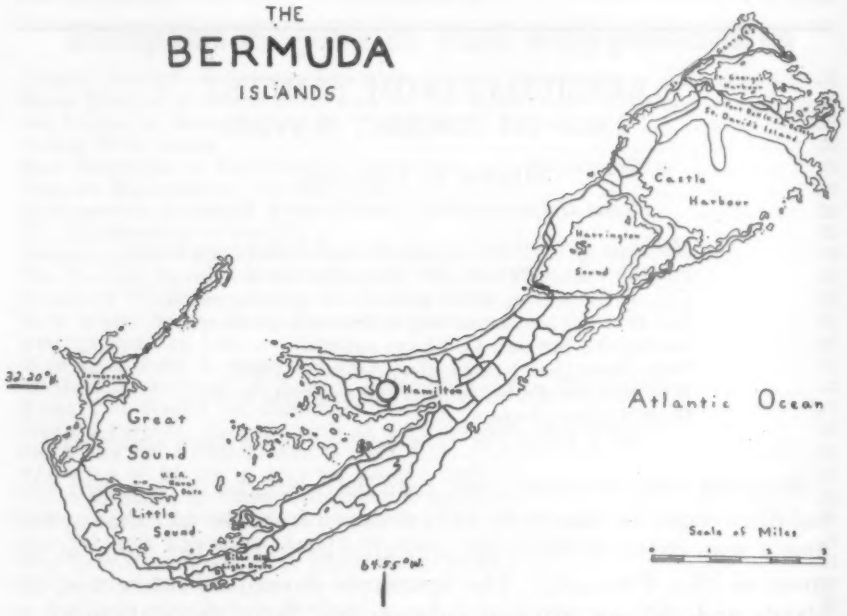

FIG. 1. The Bermuda Islands, located at 32.20°N. and 64.55°W. in the Atlantic Ocean. (Map by Betsey Rapp.)

These islands at one time covered much more area than they do today. Subsidence has taken place and only some 19.3 square miles of land stand above water. The reefs that surround the present land act as a great barrier to the ocean swell and protect the waterways in the region, all of which is extremely beneficial to the ocean vessels plying these waters and especially to the amateur sailors who enjoy sailing. This also means that there is little undertow off the beaches, making for safe swimming. It should also be added that hidden reefs, not carefully studied on the admiralty charts, may be the cause of shipwrecks. No spot on the islands is very high, the high-

est elevation being at the location of Gibb's Hill Lighthouse, 286′ (Figure 2).

Bermuda, because of its marine location, enjoys a climate that shows little variation diurnally or seasonally. The average daily range is 11.1°F. and the average monthly range is from 62.7°F. in February to 80.2°F. in August, with the average for the year being 70.7°F. Frosts are not common and have been recorded only a few times. The relative humidity, likewise, holds steady between 80 and 85 per cent. Precipitation, averaging 56.09″ per year, is fairly well distributed throughout the year but shows a minimum in the summer and often summer droughts are experienced. Rain is mostly of the showery type and of short duration and, therefore, overcast days average only 55 per year with the average number of hours of bright sunshine daily 7, ranging from 5 in December to 9.9 in July. The average velocity of the wind has been recorded as 12 miles per hour, thus there is almost always a breeze which comes mostly from the south. Inasmuch as it is off the water the breeze tends to moderate temperatures. A combination of a low temperature (for Bermuda), cloudy skies, drizzle or rain, and a normal speed breeze can make a day most uncomfortable and spoil the holiday pleasure and should be expected by the tourist in the winter and spring season. Bermuda does not lie in the major path of the hurricanes but, in the 21 year period from 1926–1947, three storms of hurricane force gave vent to their fury here, all coming in the third week of October.

Bermuda is strategically located as a meteorological base because of its Atlantic setting (Figure 3). The data collected here, both by the official Bermuda weather station and the United States Air Force weather squadron, are sent out daily for use of the ships and aircraft plying the waters and the airways of the Atlantic Ocean.

Water supply presents a tremendous problem to the people of Bermuda. Because of the porosity of the limestone, rain water does not collect on the surface of the land but seeps into the underground. Though some water is obtained by drilling wells down into the hillsides and then horizontally in different directions, it is not very satisfactory for drinking and is limited in quantity. In many places water is pumped from the sea and is used especially for sewerage purposes. Of course the most common and eye-catching sight in Bermuda comes with the slanting white roofs on which the rain water falls and then runs into gutters from which drain pipes lead to "tanks" or cisterns located either beside or beneath the houses. These tanks vary in size, holding generally from 15,000 to 25,000 gallons, but often up to 100,000 gallons. In order to keep these tanks free of mosquito larvae the Department of Public Health will provide gold fish or guppies to be placed in the water. In some areas hillside slopes may be whitewashed

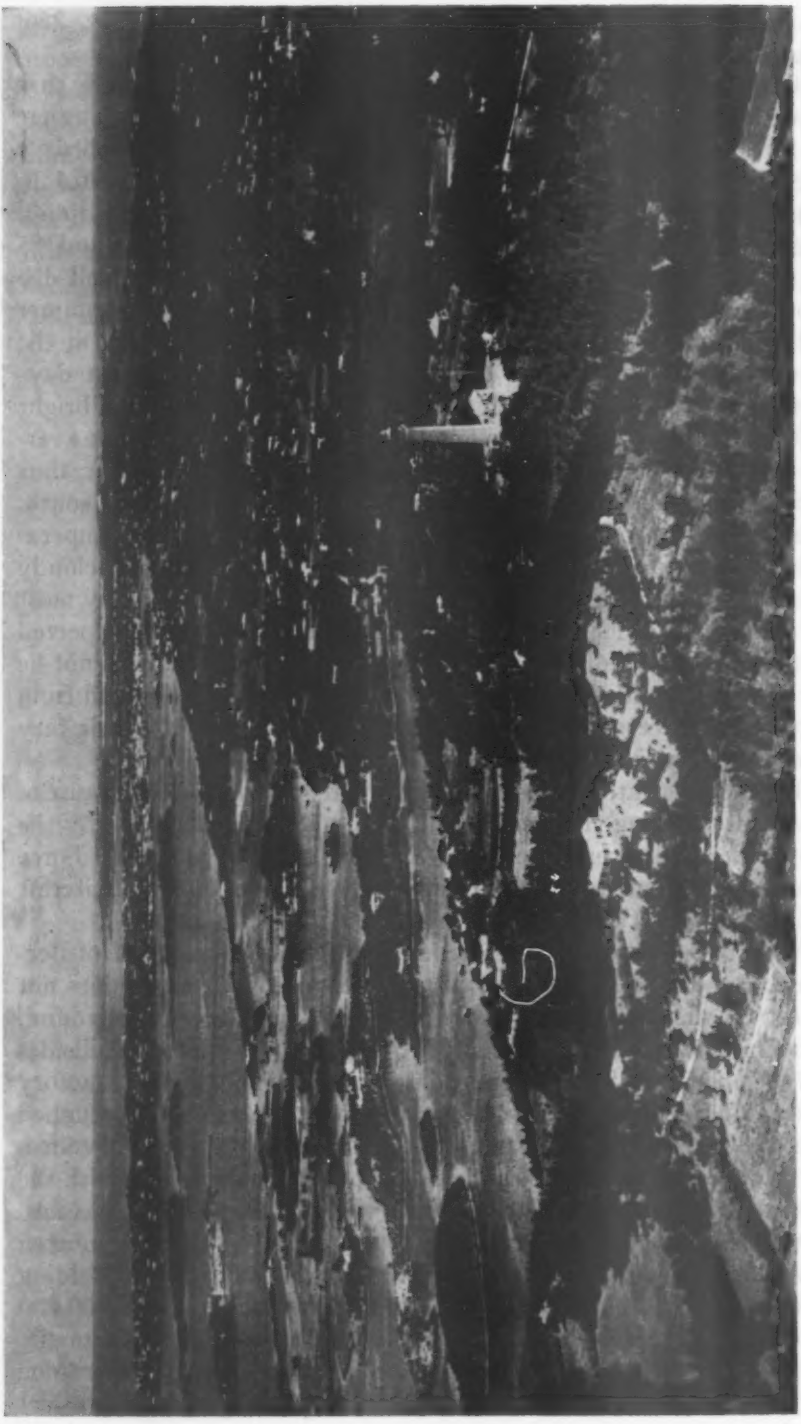

FIG. 2. Gibb's Hill Lighthouse from the air. Note the many small islands, the small sheltered bays and beaches, and the cedar trees. (Courtesy Bermuda News Bureau.)

for water catchment. One can easily appreciate the great concern of a Bermudian when summer drought or below average rainfall appear on the scene. Potable water is absolutely essential to maintain life and is one of the most precious and sought-after resources.

The native vegetation consisted mostly of mangrove along the shore line, cedar (really a juniper) and palmettoes. The climate, however, lends itself well to luxuriant growth and heavy flowering and today the chief flowering shrubs are the oleander (of which there are 40 varieties) along with the hibiscus (300 varieties) while the best known flowering plants are the Easter Lilies and the Passion Flower. The native cedar is light and durable and made good sailing vessels so that shipbuilding was an early industry. Legal restrictions have been

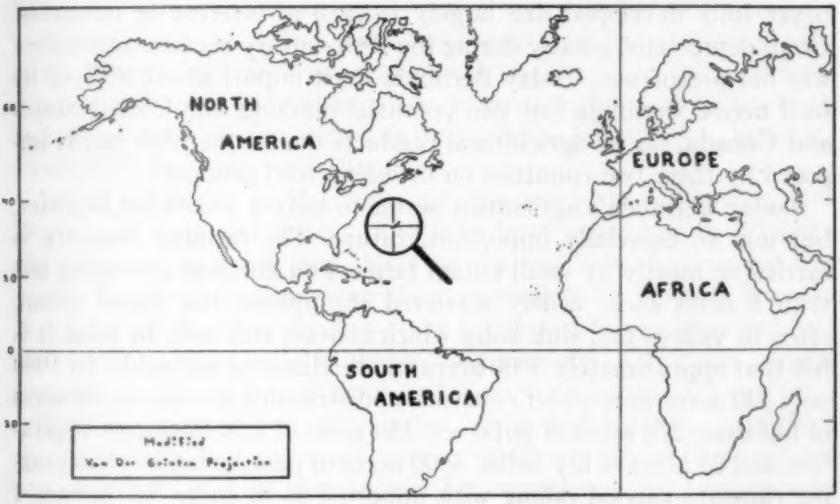

FIG. 3. Note strategic location of Bermuda in the Atlantic setting, as well as its accessibility to the United States. (Map by Betsey Rapp.)

made many times against the reckless cutting of cedar but to no avail. The cedar is resistant to salt spray and so was a protective covering for the land. Excess cutting has also led to increased soil erosion. Since 1945 a scale has attacked the indigenous cedar tree, killing many and causing much alarm. Ladybug beetles were imported in an attempt to control the scale but this method did not prove effective. Various means for checking destruction are being studied and tried along with a rather active program of reforestation. Without such trees and vegetation certainly the landscape would prove most monotonous and unattractive.

Bermuda's population averages close to 2000 people per square mile, a high density. The estimates for population for 1950 give the

following statistics: 13,497 white and 23,897 colored for a total of 37,394. The whites descend mostly from English stock while the colored people descend from slaves, many of whom intermarried with Pequod Indians who were also held as slaves.

Bermuda's climate and soils lend themselves well to agricultural production but economic factors, especially the tourist industry which requires much labor and pays good wages, work against the full development of agriculture. During the early period of settlement, tobacco and corn constituted the most important commercial crops for over 100 years. Many crops were tried in the early days, some of which were mulberry, castor oil, olive oil, indigo, saffron, pineapples, figs, pawpaws, sugar cane, plantains and pomegranates. Many of the introduced plants and trees grew and produced well but were never fully developed due largely to lack of interest or initiative. Crops depreciated greatly during the 18th century because agriculture was not prosperous. Today Bermuda must import about 90% of its food needs. Bermuda lost two potential markets, the United States and Canada, for its agricultural products due to the high tariffs imposed by these two countries on incoming food products.

Today Bermuda's agriculture seems to have a somewhat brighter, but not an especially important, future. The farming industry is carried on mostly by small tenant farmers on holdings averaging less than 5 acres each, widely scattered throughout the island group, often in valleys and sink holes which contain rich soil. In total it is felt that approximately 3000 acres can be classified as arable. In 1950 only 730 acres were under cultivation, distributed as follows: 80 acres of bananas, 275 acres of potatoes, 350 acres of miscellaneous vegetables and 25 acres of lily bulbs. 3000 acres of pasture land of low grazing capacity carried (along with imported feed) some 240 horses, 4 asses, 1200 dairy cattle, 400 goats, 2000 swine and 50,000 poultry. It is doubtful that the animal industry will show any gain. Most and best efforts seem tied up with fruit and vegetable production. Up until recently Bermuda supplied most of its vegetable needs (carrots, beets, cabbage, celery, lettuce, lima beans, spinach, kale, potatoes, onions, tomatoes) from November to May. With the installation of deep freeze equipment it is expected that the islands will provide all of the vegetable needs. Because of the frost free climate 2 and 3 crops a year can and should be grown. A wholesaling market center, started in 1949, contracts with some 60% of the farmers, helps them to plan and guarantees prices. Two and one half years after the initiation of this plan, production had increased over 250%. The agricultural department is also trying to resuscitate the citrus fruit industry. In 1952 about 80 acres were producing. The citrus association de-

partment services trees for a small fee and imports trees, selling them at cost or below cost. The orange tree has proved somewhat difficult to manage because it suffers from wind, but grapefruit and lemons stand up much better.

The agricultural industry that attracts the most interest for the tourist, however, is the flower industry. In 1950 Bermuda exported 1158 cases of lily bulbs (Easter Lily), 49 cases of miscellaneous bulbs, and 13,259 boxes of cut-flowers. Planting of Easter Lilies began about 1880 and it is General Hastings, a retired United States army veteran, who is generally credited with starting this profitable and stable industry. It certainly is a grand sight to see field after field of these lilies in blossom just at the Easter season. Even more beautiful, though, is the very delicate and magnificent Passion Flower with its five white and five purple petals.

A small perfume industry has developed from this specialty. One of the two perfume factories employs about 16 people, raises its own flowers, and takes at least one year to process the essence. Greatest interest centers on the following four plants, Easter Lily, Jasmine, Passion Flower and Sweet Pea.

Other industries in Bermuda, based upon local production, are limited. Commercial fishing to supply local consumption occupies the attention of a few men. Most of the buildings in Bermuda are made of local stone. Quarrying of limestone is relatively easy because the stone is soft. Once the limestone is exposed to the air it hardens and makes a most serviceable building material. There are, also, a few small boat building stations and a naval dockyard at Somerset.

Tourists, mostly from the United States and Canada, constitute the most important economic activity of the colony and account for approximately 90% of the revenue. The tourist industry began about 1850 when the islands were visited during the winter season as a refuge from the colder continental area to the west. During the winter of 1874 a fortnightly passenger service by steamers was begun between Bermuda and New York. Now, however, the industry flourishes as a year round industry and is constantly expanding. Cruise ships and steamers used to bring in all the visitors but now the airplane is transporting the largest percentage of the people. Chief attraction is, of course, the mild climate. For recreation, once the islands are reached, something exists to suit almost everyone's taste. Fishing, with some 269 species of fish, probably holds first place for the men. In addition, boating, tennis and shopping rate highly. (In regard to the latter may I remark that this article is being written as the author is wearing a woolen shirt, purchased in Bermuda at considerably less cost than if purchased in the United States.) Gibb's Hill Lighthouse

FIG. 4. Hamilton, Bermuda. The ocean vessel, the ferry, the sailboat and the yacht all find excellent facilities in the harbor. The white roofs certainly are conspicuous in the city and the countryside. (Courtesy Bermuda News Bureau.)

provides a splendid panoramic view of the whole area and in 1951 over 15,000 visitors were registered there.

In 1950 the tourist trade ran to about 70,000 visitors whereas the 1951 figures rose almost one-third to approximately 95,000. The length of stay of tourists has averaged, lately, 8–9 days. In the past Bermuda attracted, and still does in a large measure, what might be termed the luxury trade. The emphasis seems to be shifting somewhat now toward less expensive accommodations. Smaller hotels, boarding houses and cabins are receiving attention. In 1951 the hotels, guest houses and apartments could accommodate about 3500 guests per night. In addition to this private homes could handle approximately 500 paying guests. Accommodations, even at the peak season of Easter, can generally be found for all who desire to visit the islands; they may not be exactly what a person prefers but they are generally adequate.

In 1950, 61,654 of the 67,611 people arriving by regularly scheduled steamships or aircraft came from the United States, 3381 from Canada, 1162 from Great Britain, 1191 from the West Indies and 223 from other countries. Thus the almost complete dependence on the United States citizen shows up as a most obvious fact. The revenue accruing to this oldest British Colony in 1950 probably exceeded 15 million dollars, a substantial earning power for the small population involved, over $400 for every man, woman and child.

A comprehensive government road system of about 200 miles, plus many bridges, tie together the most inhabited islands of the Bermuda group. Horses and carriages have been the major means of land transportation and are still a different and interesting means of conveyance for the tourist. However, this represents an expensive mode because feed for the animals has to be imported and is costly. Cars were finally admitted to Bermuda in 1946 under restriction. Only "residents" or visitors who have been in the islands over 30 days are permitted to drive a car and no family is permitted to own more than one car. A railroad opened in 1931 by English investors was abandoned in 1947. Ferries operate out of Hamilton and in Great Sound connecting the western island groups, as well as in St. George's Harbour connecting St. David's and St. George's Islands.

Contact with the outside world plays an essential role in the maintenance of the economy in Bermuda. In 1950, 26,383 passengers landed from steamships operating on regularly scheduled services and 3649 arrived by cruise ships. The major scheduled carriers are the Furness Bermuda Line, Canadian National Steamships and the Alcoa Steamship Corporation. These vessels are also important in the mail service connecting the Colony with the United States, Canada and the West Indies. Mail service is also maintained by

steamship with the United Kingdom. The two principal ports are those at Hamilton (Figure 4) and St. George. Both ports have a sufficient depth for large sized vessels to enter and both have good wharfs and docking facilities. Hamilton also possesses good drydock facilities for servicing vessels, either commercial or naval. Of the vessels cleared in 1950, 223 were entered at Hamilton and 79 at St. George's.

The most spectacular growth in traffic has come with greatly increased air transport service. In 1950, 2441 craft landed in Bermuda bringing in 41,026 local passengers for disembarking and carrying 26,531 passengers in transit. Of the aircraft entering 689 were listed as British and 1752 as foreign. The major scheduled airlines are British Overseas Airways, Trans-Canada Airlines, Colonial Airlines, Cubana de Aviacion, and the largest carrier Pan American World Airways. Only 677 miles and 3 hours distant from New York, Bermuda has easy access to one of the most densely populated metropolitan districts of the world, having a high earning capacity with resultant high sums to spend for leisure and recreation.

Bermuda's trade reflects its tourist industry in its imports, especially as manufactured goods soar way beyond that generally considered normal for the size of the resident population. Three countries account for the major portion of the imports, the traditional leader the United States, closely followed by the United Kingdom and Canada. Other areas servicing the islands are Australia, the Dutch West Indies (almost entirely petroleum products), British West Indies and France. Colony produced exports have a low total value. On the other hand, Bermuda acts somewhat as an entrepot trader for British and some other foreign goods. About one-half of the trade is listed as "bunkers." Revenues from duties imposed upon imports account for practically all of the Bermuda government's revenue so there are no income taxes and other such revenue sources.

The declared value of exports in 1950 was £36,986 of which cut flowers made up £30,736, lily bulbs £3686 and vegetables £1695. The remainder of the items showed a value of less than £500 each.

In the list of general imports for the year 1950, the following ten are given in order of their value: gasoline, electrical supplies, cotton clothing, woolen clothing, fresh beef, furniture, motor vehicles, boots and shoes, whiskey, rayon clothing. Although food products, except for fresh beef, do not figure high in this grouping, they are imported in large quantities. Important items in this group are butter, eggs, Irish potatoes, canned or bottled meats, fresh pork, fruit (canned, bottled, frosted) and vegetables (canned, bottled, frosted).

Much concern is felt in Bermuda because of the relatively large per cent of land area used for military purposes. These areal holdings

occupy land which would be extremely useful for expansion by the increasing population and tourist trade. Bases not in use and empty buildings seem a waste considering the pressure on land and living quarters. At least 2.2 square miles, over 10% of the total area, are owned and occupied by Naval, Military and Air Force Departments (both British and United States). The United States Naval station and the United States Air Force Base at Kindley Field, part of which is used as the commercial base, contribute a significant sum annually to Bermuda's revenue—the estimate for 1950 being $4,000,000. Bases were leased to the United States for 99 years in 1940 as a part of the destroyer deal with great Britain. In 1941 the American flag was raised over United States Naval Bases on Morgan and Tucker's Islands.

Bermuda's peripheral location of the United States, within easy reach of large population centers such as metropolitan New York, Washington, Philadelphia and Boston, gives it an unique advantage as a tourist resort. Its central location in the Atlantic Ocean adds to this advantage, especially from the climatic point of view in that equable and mild conditions prevail throughout the year. As this climate suits the tourist so also does it favor vegetative growth. The beauty of the flowers such as the lovely reds of the hibiscus, the formal white of the Easter Lily, and the splash of the variegated colored sweet peas, provides an added attraction providing at once a sense of beauty and a sense of serenity.

1954–1955 OFFICERS OF THE NEW JERSEY SCIENCE TEACHERS ASSOCIATION

Maitland Simmons, Science teacher at Irvington High School was installed as President of the New Jersey Science Teachers Association at their annual meeting at Rutgers University on May 1st. Others elected to office include:

President-elect—Harold Hainfeld, Roosevelt School, Union City
Vice-President—Dr. Catherine Bergen, Jersey City State Teachers College
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Sister Charitas, Red Bank High School

1954 PROGRAM NOTE

MILTON O. PELLA, *Vice-President*

The program for 1954 is planned with emphasis on the teacher and teaching. The aims are reflected in a guiding statement, "Better teaching of science and mathematics for the children and youth of America." The group and sectional meetings will provide suggestions that may be helpful in improving your professional practices. The Chairmen of the several sections and groups have provided excellent leadership in securing this needed talent.

The speakers for the general sessions have been selected on the basis of their professional service to teachers. On Friday morning you will hear an address on "The Teaching of Science." The luncheon speaker will help us to better understand our complex system of communication. On Saturday morning, we are to be treated to an address on "The Teaching of Mathematics." You will note that professional as well as technical assistance is to be given to all in attendance.

The aim of the committee is the production of a program fitting the high standards demanded by the memberhsip of Central Association of Science and Mathematics Teachers. We know the meetings will warrant your attendance, and we are therefore looking forward to sharing them with you. The time is November 26 and 27, and the place is the Conrad Hilton Hotel, Chicago, Illinois. See you there.

CASMT LUNCHEON MEETING—FRIDAY, NOVEMBER 26, 1954

Dr. M. E. Strieby of the American Telephone and Telegraph Company will speak on the topic, "Memory Machines and Your Telephone." Make your reservation by completing the form below.

Please reserve _____ tickets for luncheon @ $3.35 each.

Enclosed find check or money order for $_____.

I prefer: Fish _____ Meat _____

Signed _____ Date _____

Address _____

Send reservation to Mr. Ray C. Soliday, Treasurer and Business Manager, Box 408, Oak Park, Illinois.

C. N. MILLS BEGINS WORK AT AUGUSTANA

Prof. C. N. Mills retired at the end of the summer session at Illinois State Normal University but transferred at once to Augustana College at Sioux Falls, South Dakota, where he will be in charge of classes in college mathematics. Some of our readers of a few years ago will remember Mr. Mills as our editor of the Problem Department.

A NEW TYPE STANDARD SIZE CAGE FOR RATS AND OTHER SMALL ANIMALS

JOSEPH A. KOHOUT
Department of Biology, Maryland State Teachers College, Frostburg, Md.

This type of cage was designed in our laboratory for conducting experiments with small animals. It has been used here for several years and has proven very satisfactory for raising rats, mice, guinea pigs and other small animals. The chief advantages of this cage over other homemade cages is that it lends itself to prefabrication and easy assembly. We cut and assembled 60 cages in a few hours. It is sturdy, well constructed, the materials are economically priced and easily obtained. Materials, locally purchased, cost $1.90 and this

compares very favorably with that of other well constructed but much more expensive cages.

The interior of this cage is so well protected with hardware cloth that gnawing of wood is no problem. Wild rats, while well known for their excellent gnawing ability, did no appreciable damage to the cages even though they were kept in them for a period of several weeks.

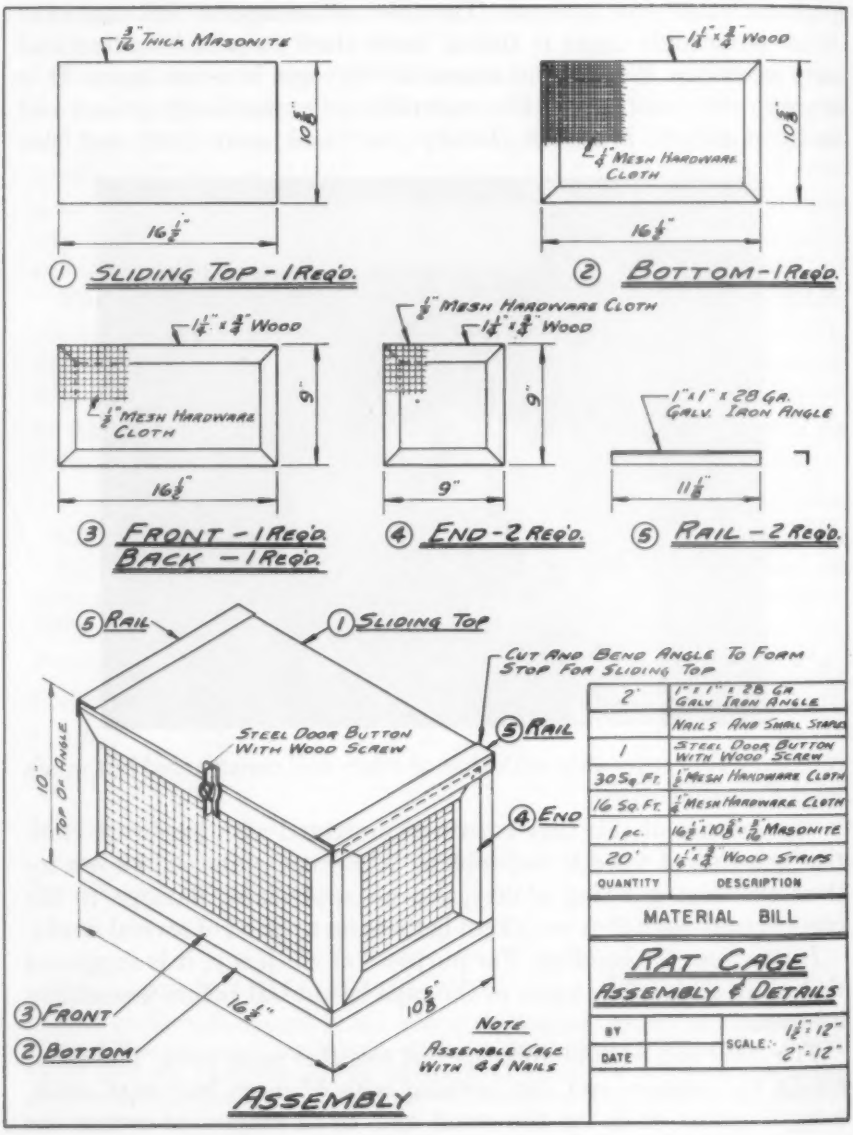
Instructions for building: For purposes of efficiency, it is suggested that all of the various parts of the cage be cut out before assembling is begun.

The sides and bottom units are put together separately. The sides, except the bottom unit, are covered with ½″ mesh hardware cloth, using 4 penny nails for the wood and small staples to secure the

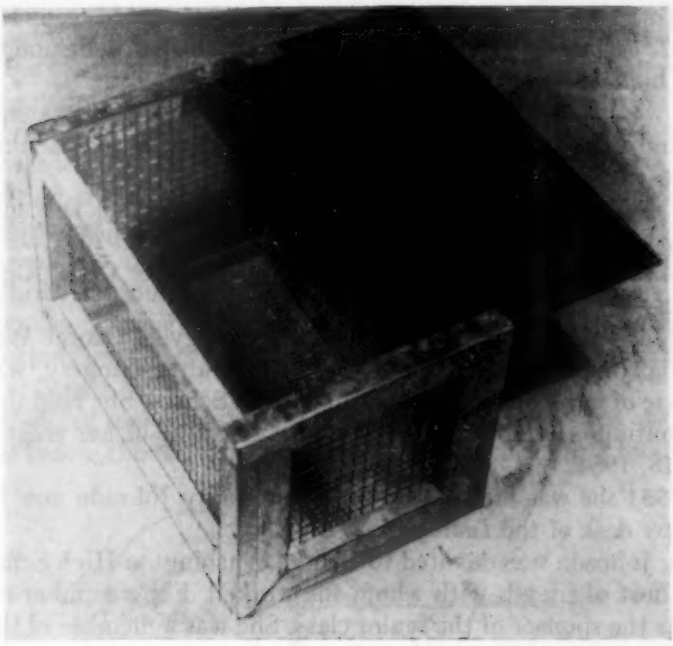
hardware cloth. The bottom unit is assembled in a similar manner but $\frac{1}{4}$" mesh hardware cloth is used.

The completed sides and bottom units are assembled as shown in the photograph and drawings. The interior of the cage is completely covered with hardware cloth to protect the wood from gnawing of the animals.

Metal angle irons are nailed to the tops of the end pieces; these form the groove in which the sliding top moves (see photograph).


3/16" THICK MASONITE
10 5/8"
16 1/2"
1 SLIDING TOP - 1 REQ'D.
1 1/4" x 3/4" WOOD
1/4" MESH HARDWARE CLOTH
2 BOTTOM - 1 REQ'D.
3 FRONT - 1 REQ'D.
BACK - 1 REQ'D.
9"
4 END - 2 REQ'D.
1" x 1" x 28 GA. GALV. IRON ANGLE
11 5/8"
5 RAIL - 2 REQ'D.
5 RAIL
1 SLIDING TOP
CUT AND BEND ANGLE TO FORM STOP FOR SLIDING TOP
STEEL DOOR BUTTON WITH WOOD SCREW
4 END
10" TOP OF ANGLE
3 FRONT
2 BOTTOM
NOTE
ASSEMBLE CAGE WITH 4d NAILS
ASSEMBLY
2" 1" x 1" x 28 GA GALV IRON ANGLE
NAILS AND SMALL STAPLES
1 STEEL DOOR BUTTON WITH WOOD SCREW
30 Sq Ft. 1/4" MESH HARDWARE CLOTH
16 Sq. Ft. 1/4" MESH HARDWARE CLOTH
1 pc. 16 1/2" x 10 5/8" x 3/16" MASONITE
20' 1 1/4" x 3/4" WOOD STRIPS
QUANTITY DESCRIPTION
MATERIAL BILL
RAT CAGE
ASSEMBLY & DETAILS
BY
DATE
SCALE: 1 1/2" = 12" 2" = 12"

A $\frac{1}{4}''$ space is allowed for the sliding top and a portion of the angle iron is bent and nailed down in the back to form a stop for the sliding top. This type of top was used because it may be completely removed and thus facilitates handling animals and cleaning the cage. It also saves on the cost of hardware because hinges, hasp and woodscrews are not needed.



A steel door button to avoid accidental opening of the cage is fastened with a wood screw as is shown in the drawing.

Our experience using 60 of these cages over a period of 5 years leads us to believe that they can be highly recommended for raising and studying small laboratory animals.

COURSES FOR STUDY OF DELINQUENCY AT UNIVERSITY OF MINNESOTA

A special academic sequence "to give students the groundwork for careers in the control of delinquency" will be offered commencing this year (1954–55) by the University of Minnesota. The 4-year undergraduate course is designed for students "who wish to prepare for graduate professional training as well as those who wish to prepare for subprofessional careers in the field." Senior college students will be given summer work placements in State institutions for boys and girls. Among the well-known professional persons in the delinquency field who will be part of the faculty for this new interdepartmental sequence are Elio. D. Monachesi, Dale Harris, Gisela Konopka, and John R. Ellingston. For a bulletin on this "Training for Delinquency Control" course, write to Dept. of Sociology, University of Minnesota, Minneapolis, Minn.

LOIS GERALDINE JOHNSON
1904–1954

Mrs. Johnson was born November 30, 1904 at Warsaw, Indiana. Her childhood years were filled with experiences that encouraged her to study languages, music and science. Her father and mother, Mr. and Mrs. Albert Reep of Indianapolis were graduated from DePauw in 1902. For many years her father was a principal and superintendent of high schools.

She was graduated from Shortridge High School, Indianapolis in 1923 and from DePauw University in 1926, completing each of these four year courses in three years.

Her mother called our attention to her activities in DePauw University by a note from "The Mirage": V. P. Class 1926, Co-Ed. basket-ball, soccer, bowling, Rifle Club, Messiah, Glee Club, Alpha Chi Omega.

In Jan. 1928 she was invited to teach biology at George Washington High School, Indianapolis, by Prin. Walter Gingery, who had been one of her teachers in Shortridge High School. She held this position continuously since that date until the time of her tragic death April 18, 1954.

In 1934 she was married to Mr. John Henry Johnson now head of the copy desk of the *Indianapolis Star*.

Mrs. Johnson was devoted to George Washington High School and had a host of friends with whom she worked. For a number of years she was the sponsor of the senior class. She was a member of the Central Association of Science and Mathematics Teachers, and for several years had been the membership chairman of that organization, and had edited its Year Book. Her active interest in the Association made her one of our most valued members.

She died as a result of severe body burns when her clothing was ignited in her home on Jan. 25, 1954.

Her many students through the years she taught in George Washington High School will remember how they were encouraged to go on into research in the fields of disease, especially the field of cancer.

Her courage and continued interest in her students and associates during her intense suffering will never be forgotten and her loss to her high school, community and relatives is deeply felt by all.

ESTIL B. VAN DORN
GLEN W. WARNER

Razor planer uses ordinary double-edge razor blades for small planing jobs such as shaving edges of narrow cabinet doors, shelves and moldings. The tiny tool can plane curved areas and is useful in model-making.

UNORTHODOX WAYS TO TRISECT A LINE SEGMENT

CHARLES W. TRIGG
Los Angeles City College, Los Angeles, Calif.

Five methods for trisecting a line segment are offered here in the hope that they may stimulate some of the better geometry students to prove the constructions and to generalize the procedures to n-secting.

To provide a means of comparison of the methods, we use a geometrographic index which is the total of the operations performed in the construction. Starting with closed compasses, opening to a particular setting is counted as one operation, as is changing the compasses to another setting. Drawing a straight line, describing a circle or striking an arc are each counted as one operation. In the following procedures, the number of the step is indicated by the number in parentheses. Throughout, (A) indicates the circle with center at A. The dotted lines in the figures are suggestions toward proofs of the constructions.

1. *Modified conventional method.* In order to reduce its geometrographic index, the conventional method has been slightly modified.

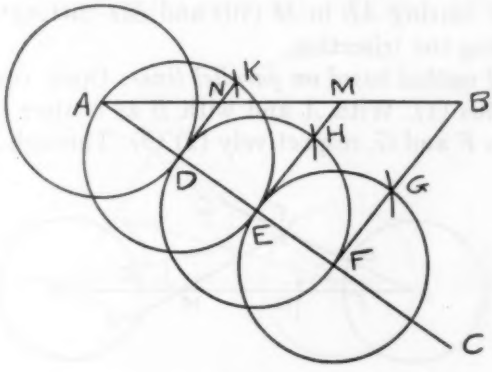


FIG. 1

Given the line segment AB. Through A draw line AC making an arbitrary acute angle with AB (1). Open compasses to an arbitrary radius (2). With A as center describe circle cutting AC in D (3). With D as center describe circle cutting AC in E (4). With E as center describe circle cutting AC in F (5). With F as center describe circle (6). Draw FB cutting circle (F) in G (7). Set compasses to radius equal to EG (8). With D as center describe arc cutting circle (E) in H (9). With A as center describe arc cutting circle (D) in K (10). Draw EH

cutting AB in M (11). Draw DK cutting AB in N (12). This completes the trisection of AB.

2. *Method based on parallel lines.* (Called to the writer's attention by Fred Marer.) Open compasses to an arbitrary radius (1). With A and with B as centers describe circles cutting AB in C and D, respectively (2) (3). With C and D as centers describe arcs cutting (A) and (B), respectively in points E and F on opposite sides of AB (4) (5).

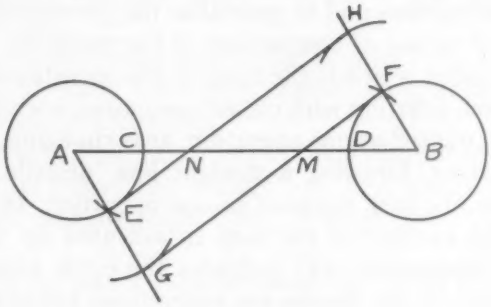


FIG. 2

Draw AE extended and BF extended (6) (7). With E and F as centers describe arcs cutting AE and BF in G and H, respectively (8) (9). Draw GF cutting AB in M (10) and EH cutting AB in N (11), thus completing the trisection.

3. *Modified method based on parallel lines.* Open compasses to an arbitrary radius (1). With A and with B as centers describe circles cutting AB in F and G, respectively (2) (3). Through A draw AC at

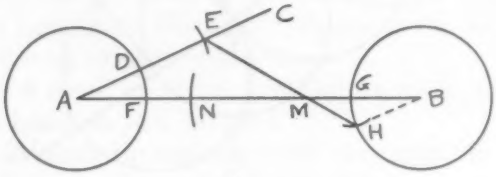


FIG. 3

an arbitrary angle to AB and cutting circle (A) in D (4). With D as center describe arc cutting AC in E (5). Change opening of compasses to radius equal to DF (6). With G as center describe arc cutting circle (B) on the opposite side of AB from E in H (7). Draw EH cutting AB in M (8). Set compasses to radius equal to BM (9). With M as center describe arc cutting AB in N, thus trisecting AB (10).

4. *Method based on a square.* Open compasses to radius greater than $\frac{1}{2}AB$ (1). With A and B as centers describe arcs intersecting in

C and D (2)(3). Draw line through CD cutting AB in E (4). Change opening of compasses to radius equal to AE (5). With E as center describe circle cutting CD in F and L (6). With F as center describe arc cutting CD in G (7). With G as center describe circle (8). Draw BF

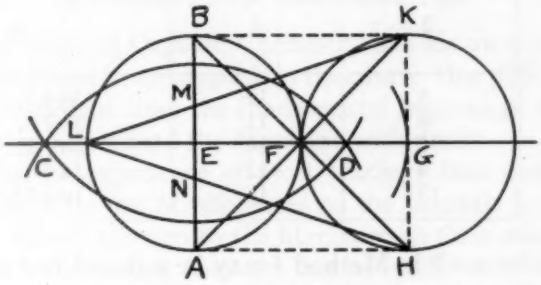


FIG. 4

meeting (G) in H (9). Draw AF meeting (G) in K (10). Draw KL and HL cutting AB in M and N, respectively, thus trisecting AB (11)(12).

5. *Method based on Ceva's Theorem.* Extend AB in both directions (1). Open compasses to an arbitrary radius greater than $AB/5$ and less than AB (2). With A and B as centers describe circles cutting BA extended in C, and AB extended in D (3)(4). With C and D as centers describe arcs cutting BA extended in E, and AB extended in F

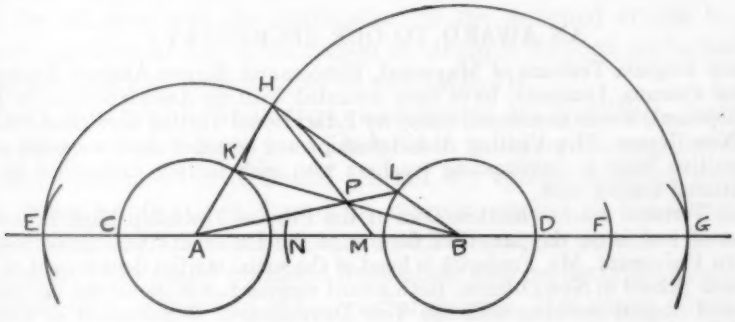


FIG. 5

(5)(6). With F as center describe arc cutting AB extended in G (7). Change compasses to radius equal to AE and with A as center describe circle (8)(9). Change compasses to radius equal to BG and with B as center describe circle intersecting large circle (A) in H (10)(11). Draw AH meeting small circle (A) in K (12). Draw BH meeting small circle (B) in L (13). Draw BK and AL meeting in P (14)(15). Draw HP meeting AB in M (16). Change compasses to radius equal to MB and with M as center describe arc cutting AB in N (17)(18).

Generalizations. The values assumed by the geometrographic index when the five methods are generalized to n-secting are given in the body of the following table for various values of n.

Method \ n	2	3	n
1	9	12	$3n+3,\ n>1$
2	6	11	$3n+2,\ n>2$
3	7	10	$2n+4,\ n>2$
4	4	12	$3n+4,\ n>3$
5	13	18	$2n+12,\ n>2$

The index for $n=2$ in Method 3 may be reduced to 6 by retaining the same radius after drawing (A) and (B) and then striking arcs with F and G as centers. The join of the intersections of these arcs with (A) and (B) bisects AB.

In the generalization of Method 4 the square becomes a rectangle.

Care must be taken in Method 5 that the initial opening, r, of the compasses be such that $AB/(n+2) < r < AB/(n-2)$.

For $n>2$, Method 3 is always the most efficient one of the five methods. When $n>9$, Method 5 moves up to second place in the order of efficiency.

AN AWARD TO OUR SECRETARY

Mary Virginia Terhune of Maywood, Illinois, and Wayne Andrew Frederick of New Orleans, Louisana, have been awarded Visiting Associateships in Test Development, it was announced today by Educational Testing Service of Princeton, New Jersey. The Visiting Associateships are awarded each summer on a competitive basis to outstanding teachers who wish further experience in the educational testing field.

Miss Terhune teaches mathematics at the Proviso Township High School in Maywood and is on the part-time faculty of the University College of Northwestern University. Mr. Frederick is head of the social studies department of the Newman School in New Orleans. Both award recipients will spend the months of July and August working with the Test Development Department of Educational Testing Service in Princeton.

Educational Testing Service, a nationwide, non-profit organization, carries on a wide variety of educational testing and research activities including the preparation and administration of tests for the Selective Service System, the College Board, the Naval Reserve Officers Training Program, and many other governmental and educational agencies. Its main officers are in Princeton, New Jersey, and Los Angeles, California.

Electrical repair kit includes everything needed for simple electrical work. A pliers-like tool cuts and strips wire, crimps solderless terminals and connectors, and shears bolts and screws. Splices, terminals, connectors and wire insulation are included in the kit.

SUGGESTIONS FOR TEACHING ORGANIC CHEMISTRY

ROBERT H. MITCHELL
Muskingum College, New Concord, Ohio

Every instructor of Organic Chemistry has his own list of objectives which he hopes to accomplish in the course. One objective might be to lead the student from the fundamental beginnings to the point where he can and does read the literature with profit.

It has been the experience of most teachers that undergraduate students make little use of references to the journals found in text books, much less do they go to the literature on their own initiative. This does not indicate that they are lazy or indifferent, but rather, I believe, that they lack motivation. Perhaps they are frightened by the maze of organic formulae and terminology and see no real reason why they should labor through an uninteresting article simply because the author of the text or the instructor thinks they should.

If the instructor keeps clearly in mind the fact that some students take a course in Organic Chemistry as part of a more extensive study of chemistry while others take it in preparation for medicine, dentistry, agriculture, pharmacy, etc., he may use these vocational goals of the student to motivate the reading of source material. Outside reading assignments may then be made on the basis of interest.

If the student sees the application of the assigned article to his chosen vocation he has more reason to struggle through its technical detail and make an honest effort to understand it.

Many worth while industrial publications are available, just for the asking, which I have found helpful in bridging the gap between the text book and the technical journal. By special assigned reports from these publications, based on interest appeal to the assignee, students get their first real contact with literature outside the text. Information not easily obtained from other sources becomes available to the student in the field of his special interest. If it seems advisable to the instructor this material may be made available to the entire class through oral reports and discussion.

Finding the assigned material of interest to him vocationally the student's resistance to the literature has been somewhat softened and eventually may be broken down completely. Obviously the next step is the assignment of an article in a journal, again based on the student's vocational interest.

A Journal Club is another means by which students may be introduced to the technical literature. The Journal Club may be made up of organic students only or it may be open to all students in advanced

courses. While the exact make up and working of the organization may vary, it is essentially a group of students and instructors who meet periodically to review and discuss the current literature of the subject matter field. Here again vocational interest should be considered in assigning articles for review. A club specializing in Organic Chemistry has advantages for the organic student since a smaller group gives each student the opportunity of making more reports and, therefore, gives him more contacts with the literature. The field of subject matter discussed is not so diversified as must be the case in a Chemistry Journal Club open to all advanced chemistry students. If only one Journal Club is practical for the Chemistry Department I would not favor limiting it to organic students only because I believe that the Journal Club can be of great value to advanced students in any field.

Properly used the bulletin board may stimulate interest and be a source of considerable information. I believe that timely news items concerning chemistry and related fields are of definite educational value to the students. A bulletin board, near the class room or laboratory, set aside for items of interest to the student of Organic Chemistry may be a helpful teaching device. The value of this device is determined by the type of items placed on the board and the use students and instructors make of it.

Experienced teachers know that facts are retained better when they are coupled with an anecdote or some interesting information within the student's field of interest and experience. The bulletin board may serve the purpose of presenting timely, interesting facts and bits of information by which the student may "clinch" the material learned from the text and lecture.

Weekly news magazines, newspapers and popular magazines often carry items, notes and stories of interest to the student of Organic Chemistry. Through these items the student may be made conscious of the practical applications of the things learned to every day life.

Students should be strongly urged to bring in items of interest for posting and should be made to feel that they have made a significant contribution when they respond. This serves to keep the student alert to watch for applications and to see the practical value of Organic Chemistry in the world of today.

The bulletin board may be used: 1) to stimulate student interest, 2) to make him aware of the practical value of Organic Chemistry, 3) to keep the student alert to the opportunities of Organic Chemistry, 4) as a point of departure in introducing a new topic for discussion, 5) as a means of "clinching" material already learned, and 6) for raising of questions.

The advertising of some chemical and related manufacturing organizations, found in the technical journals, often presents interesting and useful information as to the structure of a compound, its preparation, properties, history and uses. This type of item makes a place for itself on the bulletin board. Since the dateline becomes less important such items may be saved until needed to introduce a topic or augment the class discussion. The instructor may keep a supply of such items on hand for use when more newsy materials are not available or until they are needed to augment class work.

The bulletin board may be made a permanent assignment and students held responsible for the contents of all items posted. The material should be changed frequently, no item should remain on the board more than one week, to keep the student alert. If the contents of the bulletin boards are discussed frequently in class the student is encouraged to read the items and relate them to the study, thus enriching his fund of information.

In suggesting ways and means of making chemistry more interesting to the student Elbert C. Weaver[1] mentions the fact that exhibits are always interesting to students. Many interesting and instructive exhibits rich in educational value are available to chemistry instructors, and others may be prepared by the students or instructor. The use of such exhibits as teaching devices is limited only by the ingenuity of the instructor. Such exhibits may have values comparable to those suggested for the bulletin board and these often may be used to supplement each other.

I know of no better way to give the student an appreciation of the contribution of Organic Chemistry to our present civilization than a well chosen display of industrial exhibits.

Industry has made great contribution to the field of Organic Chemistry in its efforts to learn and apply the principles and facts of Organic Chemistry to practical uses. No one need apologize for making Organic Chemistry of practical value to the student. Keeping the student conscious of the practical values and applications of the material studied is, I believe, a good teaching device to be used freely in motivating the student. The bulletin board, the industrial publications and exhibits all make contribution to this end. Practicality need not and must not replace thoroughness in mastery of principles and essential facts.

In teaching Organic Chemistry it is desirable that the student be led to understand clearly the important relationships between compounds and reactions. It is important that the continuity of the subject be impressed upon him.

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[1] Weaver, Elbert C., *Journal of Chem. Ed.*, Vol. 27, No. 10, p. 536 (Oct., 1950).

Schematic charts are of value in showing the continuity of the subject by indicating the relationship of compounds and reactions. They are of value in summarizing the work and are useful in review. Such charts are found in most texts, some covering the material discussed in one chapter only and others covering larger areas of the field. Charts covering the whole area of the aliphatic compounds and the area of the aromatic compounds are available.

While the value of these charts is not to be underestimated I believe that charts cooperatively prepared by students and instructor working together are of much more value as teaching devices. As the class discussion progresses the students and instructor may prepare a chart to show the relationships that exist between compounds and processes. This serves to "clinch" the study. The chart is expanded and kept up to date through the course until one chart results which not only summarizes the course but clearly indicates the continuity of the field.

If the instructor wishes he may vary this procedure by requiring each student to prepare his own chart and keep it up to date. In any case the instructor should make it his business to see that the student understands the chart, otherwise it is of not more value than charts already available. The advantage of the student prepared chart is that the student must clearly see and understand the relationships that exist in order to prepare the chart properly. Since he has prepared the chart and understands its meaning he can use it more intelligently for summarizing the work to date and for reviewing and later study. The chart then becomes a valuable study tool for the student. This is not always the case with the prepared charts, which the student often does not understand.

What has been said about the teaching of Organic Chemistry applies equally well, with minor variations, to any of the fields of chemistry.

UNIVERSITY OF SOUTHERN CALIFORNIA GIVES COURSES FOR LAW ENFORCEMENT OFFICERS

Twenty scholarships are being offered by the Delinquency Control Institute of the University of Southern California to qualified law enforcement officers from anywhere in the U. S. or abroad. The scholarships allow full tuition for a 12-week course dealing exclusively with problems in juvenile delinquency control and in the police-juvenile relationship. According to the Institute authorities, local departments usually pay an officer's regular salary while he is attending the course; living and travel expenses are also paid by the department or by civic groups. The course is offered regularly each academic term. So far as the Children's Bureau can determine, no equivalent course is offered anywhere else in the U. S. Inquiries should be addressed to the Director, Delinquency Control Institute, University of Southern California, Los Angeles 7, Calif.

THEOREMS AND FORMULAS CONCERNING FUNDAMENTAL RIGHT ANGLE TRIANGLES

ALVIN P. CORNELL
5221 Kenwood Ave., Chicago 15, Ill.

A fundamental right angle triangle (F.R.A.T.) has certain properties which make it unique among right angle triangles. These properties are such that it becomes possible to construct F.R.A.T.'s on a massive scale—by formula. In fact, any numeral from 1 to infinity (except numeral 1 itself) can form one of the two sides of a F.R.A.T. That one dimension is all that is necessary to construct quickly and easily the other side and hypotenuse.

Each F.R.A.T., in its dimensions and proportions, is unlike any other. Using any given F.R.A.T. as a base, a multitude of right triangles having identical proportions can be constructed therefrom. Such right triangles, when derived as some multiple of a F.R.A.T., are here called derivative right angle triangles (D.R.A.T.'s). The multiple is greater than unity if the dimensions of a F.R.A.T. are multiplied, less than unity if they are divided.

The symbols X, Y and Z will be used in this discussion. X will represent one side (the squared one), Y will represent the other side (the non-squared one), and Z will represent the hypotenuse. It is side X which can be represented by any numeral save the numeral 1.

Side X, in the great majority of instances, will turn out to be the short side; in some relatively few cases, however, it may turn out the to be long side. To clarify this situation, therefore, we shall divide F.R.A.T.'s. into two classes: a COMFRAT class and an INFRAT class. In the COMFRAT class, the most common by far, the X side will be the short side. In the INFRAT class, examples of which are infrequently met, the X side will be the long side.

THEOREMS

1. The square of one of the sides of a F.R.A.T. equals the sum of other side plus hypotenuse.
2. The hypotenuse of a F.R.A.T. exceeds by exactly 1 the non-squared side, whether the latter be the short or long side.
3. The sum of the dimensions of any F.R.A.T. is equal to $X+X^2$, where X equals any number but 1.
4. Any number from 1 to infinity (except number 1) can be set equal to side X and thereby enable a F.R.A.T. to be formed.
5. Any multiple of a F.R.A.T. is itself a right angle triangle.
6. In a COMFRAT, the short side, squared equals sum of long side plus hypotenuse and the latter exceeds long side by exactly 1.

7. In an INFRAT, the long side, squared equals sum of short side plus hypotenuse and the latter exceeds short side by exactly 1.
8. When X, the side squared, is equal to 2.42 or any larger number, a COMFRAT will result and side X will be the short side.
9. When X, the side squared, is equal to 2.41 or any smaller number, an INFRAT will result and side X will be the long side.

Formulas

1. In any F.R.A.T., when one side or X equals any numeral except 1, then:

(a) $$Y=\frac{X^2-1}{2}$$

(b) $$Z=\frac{X^2+1}{2}.$$

2. In any COMFRAT, when one side or X equals 2.42 or any larger number, then that will be the short side, and

(a) $$Y=\frac{X^2-1}{2}$$

(b) $$Z=\frac{X^2+1}{2}.$$

3. In any INFRAT, when one side or X equals 2.41 or any smaller number, then that will be the long side, and

(a) $$Y=\frac{X^2-1}{2}$$

(b) $$Z=\frac{X^2+1}{2}.$$

How to Determine a Right Angle Triangle by the F.R.A.T. System

Any three dimensions reputed to represent a right angle triangle can be quickly tested in a simple manner. For example, assume we are given the dimensions of 45–108–117 and asked whether they form a right triangle. The answer by steps is as follows:

Step #1

Find the multiple, which is here the difference between the hypotenuse and long side. Multiple equals 117 minus 108 or 9.

Step #2

Divide all dimensions by the multiple.

$$9 \mid \begin{array}{r} 45-108-117 \\ \hline 5-\ 12-\ 13 \end{array}$$

Step #3

If result of Step #2 is a F.R.A.T., the answer is yes; if not, the answer is no. 5, squared equals 12 plus 13, and hypotenuse exceeds long side by exactly 1. Therefore, since 5–12–13 is a F.R.A.T., the answer is yes. The 45–108–117 right triangle is a D.R.A.T., being the ninth multiple of the 5–12–13 F.R.A.T.

How to Construct F.R.A.T.'s Quickly by the F.R.A.T. Law of Progression

With one exception, a series of F.R.A.T.'s is shown in Table 1. Those formed from consecutive odd numbers representing X are to the left; those formed from consecutive even numbers representing X are to the right.

It will be noted that the figures in column X increase successively by two, whereas the figures in columns Y and Z increase at a rate equal to X plus the X immediately following. For example, the increase in Y and Z from 11–60–61 to 13–84–85 equals 24, or 11 plus 13.

Table 1

One Side X	Other Side Y	Hypotenuse Z	One Side X	Other Side Y	Hypotenuse Z
1	0	1	2	$1\frac{1}{2}$	$2\frac{1}{2}$
3	4	5	4	$7\frac{1}{2}$	$8\frac{1}{2}$
5	12	13	6	$17\frac{1}{2}$	$18\frac{1}{2}$
7	24	25	8	$31\frac{1}{2}$	$32\frac{1}{2}$
9	40	41	10	$49\frac{1}{2}$	$50\frac{1}{2}$
11	60	61	12	$71\frac{1}{2}$	$72\frac{1}{2}$
13	84	85	14	$97\frac{1}{2}$	$98\frac{1}{2}$
15	112	113	16	$127\frac{1}{2}$	$128\frac{1}{2}$
17	144	145	18	$161\frac{1}{2}$	$162\frac{1}{2}$
19	180	181	20	$199\frac{1}{2}$	$200\frac{1}{2}$

Table 1 was constructed partly by formula and partly by the law of progression for F.R.A.T.'s. When a series of consecutive numbers is used to represent one side (X) of a series of F.R.A.T.'s, the law of progression prevails.

Let us combine in imagination the two sections of Table 1 into one table of consecutive numbers beginning with 1. It will then be noted that, although the figures in column X increase at a constant rate of 1, the figures in both columns Y and Z increase at identical rates—in a fashion roughly similar to those in Table 1.

Law of Progression

By the F.R.A.T. law of progression, when X is represented by a consecutive series of whole numbers, X will increase at a constant

rate of 1, while both Y and Z will increase at a rate equal to $\frac{1}{2}$ less than the figure representing X, as shown in Table 2.

TABLE 2

One Side X	Other Side Y	Hypotenuse Z	Increase in both Y and Z
1	0	1	—
2	$1\frac{1}{2}$	$2\frac{1}{2}$	$1\frac{1}{2}$
3	4	5	$2\frac{1}{2}$
4	$7\frac{1}{2}$	$8\frac{1}{2}$	$3\frac{1}{2}$
5			$4\frac{1}{2}$
6			$5\frac{1}{2}$

The law of progression, therefore, makes possible the construction of an indefinite table of F.R.A.T.'s without resorting to the formulas.

How to Determine Two Unknown Dimensions of Any Right Angle Triangle from Two Facts

With the F.R.A.T. method, one can determine two unknown dimensions of any right triangle from two facts concerning it. The facts needed would be 1) one side (usually the short side), and 2) the multiple.

For any right triangle, the following formulas hold true. In the formulas, X equals one of the sides, Y equals the other side (non-squared), Z equals the hypotenuse, and M equals the multiple or hypotenuse minus the non-squared side. (When, as in the example below, we are given the dimensions of one side, the multiple is the difference between the two unknown dimensions, Y and Z.)

Formulas for Any Right Triangle

$$(1) \qquad \frac{X^2}{M} = Y + Z$$

$$(2) \qquad \frac{X^2 - M^2}{2M} = Y$$

$$(3) \qquad \frac{X^2 + M^2}{2M} = Z$$

Let's assume a right triangle with dimensions, unknown to us, of 120–225–255, or X, Y and Z respectively. If given two facts about that right triangle—one side and the multiple—it is possible to name the other two dimensions. These two facts would be 1) 120 (side X) and

2) 30 (multiple). Then, by formula

$$\frac{X^2}{M}=\frac{120^2}{30}=Y+Z.$$

Since we know that Z equals $Y+M$ ($Y+30$ here), we therefore know that $Y+Z$ equals $2Y+M$ ($2Y+30$ here), and we may either solve two linear equations or use the other two formulas to obtain Y and Z.

The 120–225–255 right triangle is the 30th multiple of the 4–7½–8½ F.R.A.T.

(If, in the foregoing formula No. 1, we substitute any value for X and imagine some value equal to M, the result will be *some* right triangle—as long as our value for M is not equal to or greater than that of X).

How to Make Your Choice of a Right Triangle

Let's imagine that you want to custom fit at least one side of a right triangle to fit your needs, and, being selective, you would like a choice among several right triangles regarding the other side and hypotenuse.

Let's assume that you want one side of 40. A simple way to meet the problem is with a technique that is shortened by telescoping the F.R.A.T. and D.R.A.T. systems.

A little reflection will tell you that 40 is an integral multiple of various numbers. Since 40 is divisible by 2, 2½, 4, 5, 8, 10 and 20, it is the 20th, 16th, 10th, 8th, 5th, 4th, and 2nd integral multiple of those numbers, respectively. This means that there are F.R.A.T.'s with side X equal to 2, 2½, 4, 5, 8, 10 and 20 which can be magnified into D.R.A.T.'s with side X equal to 40. Therefore, you already know that, aside from a F.R.A.T., there are at least seven (7) possible right triangles (D.R.A.T.'s) from which you can select. What are their dimensions?

Table 3

M or Multiple	X^2/M	$Y+Z$	$Y+Z$ equals $2Y+M$ or	$2Y$	Y	Z
20	1600/20	80	$2Y+20$	60	30	50
16	1600/16	100	$2Y+16$	84	42	58
10	1600/10	160	$2Y+10$	150	75	85
8	1600/8	200	$2Y+8$	192	96	104
5	1600/5	320	$2Y+5$	315	157½	162½
4	1600/4	400	$2Y+4$	396	198	202
2	1600/2	800	$2Y+2$	798	399	401

Recalling the formula stating $X^2/M=Y+Z$, and remembering that

Z equals $Y+M$, you can set up a table similar to Table 3 and quickly arrive at each of the other two dimensions (Y and Z), knowing that one side in each right triangle will be 40. You are then free to make your choice.

In only one case is Y (30) smaller than X (40). The reason for that is the 40-30-50 right triangle is the 20th multiple of the $2\text{–}1\frac{1}{2}\text{–}2\frac{1}{2}$ F.R.A.T., which is an INFRAT inasmuch as the long side, squared equals short side plus hypotenuse. All the other right triangles are multiples of some COMFRAT.

1955 PROGRAM OF RECOGNITION AWARDS FOR SCIENCE TEACHERS

Elementary and secondary science teachers are again encouraged to report their "best new ideas of the year." The Future Scientists of America Foundation of the National Science Teachers Association announces the 1955 Program of Recognition Awards for Science Teachers. This program, now in its fourth year, is sponsored by the American Society for Metals and is a cooperative attempt to recognize science teachers who develop good ideas and are willing to share them with other teachers. Complete information may be obtained from the Future Scientists of America Foundation, 1201 Sixteenth Street, N.W., Washington 6, D. C.

In 1954 the first place award of $400 went to Edward Victor, head of the science department of Rogers High School, Newport, Rhode Island. With the aid of his fellow teachers, Victor was able to analyze the study troubles which his students encountered and to develop teaching methods which help students improve their study habits.

Maurice Bleifeld of Newton High School, Elmhurst, New York reported how he has used the discoveries and inventions of great scientists to show his students how scientists approach and solve problems. He received the second place $300 award.

Phyllis Busch, biology teacher in Abraham Lincoln High School in Brooklyn, New York, was recognized with the $200 third place award for her story on how she converted a vacant city lot into a source of field experience thought to be available only to children in suburban or country schools.

Stanley C. Pearson received fourth place award of $100 for his idea which featured easily constructed paper models that help his students to make sense out of abstract mathematical concepts. He devised paper gadgets that helped him teach many of the subjects from physics that ordinarily require elaborate equipment. The old problem of determining the meeting point of two vehicles which start from opposite ends of a road and proceed toward each other at different rates of speed can be solved by his "rapid grapher" made by using tagboard and colored yarns. Quoting Pearson, "The number of audio-visual aids that can be created from paper seems to be limited only by the amount of cutting, folding, and gluing that a person is willing to do."

Word-game deck is a playing-card version of popular word and spelling games that have swept the country. Conventional rummy and solitaire may also be played with the 52-card deck. Instructions give up-to-date rules for each game. A second deck for children has simplified, easy-to-understand rules. For further information write to Educational Cards Inc., Industrial Bank Bldg., Detroit, Mich.

TRIGONOMETRY AND THE PRACTICAL DESCRIPTIVE GEOMETRY OF OBLIQUE PLANES

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University of Toronto, Toronto, Canada

(1) In the course of experiments on the rocking of bodies[1] I became interested in the rocking of a leaning ring. Imagine a thin metal ring standing, edge-on, upon a horizontal table and leaning against a vertical wall. If one side be pushed in towards the wall and then released the ring rocks to-and-fro about its equilibrium position. For a given angle of inclination of the plane of the ring what is the period of the rocking movement? How does the period vary with the inclination of the plane of the ring to the horizontal? How does it vary with the size of the ring?

As far as I know this is a hitherto unsolved problem. In the course of my investigation I was led to consider the properties of oblique planes, i.e., planes inclined to both the horizontal and vertical coordinate planes, and to link the diagrams obtained by Descriptive Geometry with the trigonometrical functions of the angles involved.[2] My results may be of interest to others.

For the benefit of a beginner in Descriptive Geometry, I open with a short description of the methods employed in that subject. There are of course many textbooks dealing with the practical side of Descriptive Geometry. The one I used at school in England over 60 years ago and which I still find most helpful is Practical Plane and Solid Geometry by T. W. Good (G. Gill, London, 1890).

(2) In the simplest ordinary descriptive geometry of solids we imagine an object standing on a horizontal plane (HP for short) in front of a vertical plane (VP for short). The intersection of the HP and the VP is called the ground line (denoted by GL or XY). The plan of the object is what we see when we look straight downwards on it (from a great distance) and the front elevation is what we see when we look horizontally at the object (from a great distance). For example, Fig. 1a shows a right circular cone standing on the HP. The plan is a circle with a dot in the center and the elevation an isosceles triangle. In descriptive geometry we are said to *project* the points and lines of the body upon the HP and VP and then we revolve or "rabatte" the VP back around XY as a hinge until it lies horizontally so that plan and elevation may be shown on the horizontal sheet of paper. Thus Fig. 1b shows the plan and elevation of the cone of Fig. 1a.

[1] *American Journal of Physics*, Vol. 18, 1950, Oct., p. 405; Vol. 19, 1951, Dec., p. 511; Vol. 20, 1952, Sept., p. 338; Vol. 21, 1953, April, p. 267.

[2] Some of these relations will also be found in the textbooks of Spherical Trigonometry.

Important points are marked with undashed letters in the plan and with the same letters dashed in the elevation. Capital letters are used sometimes to show the real points out in space as in Fig. 1a or in the true shapes of bodies. Consistency of use is not always feasible.

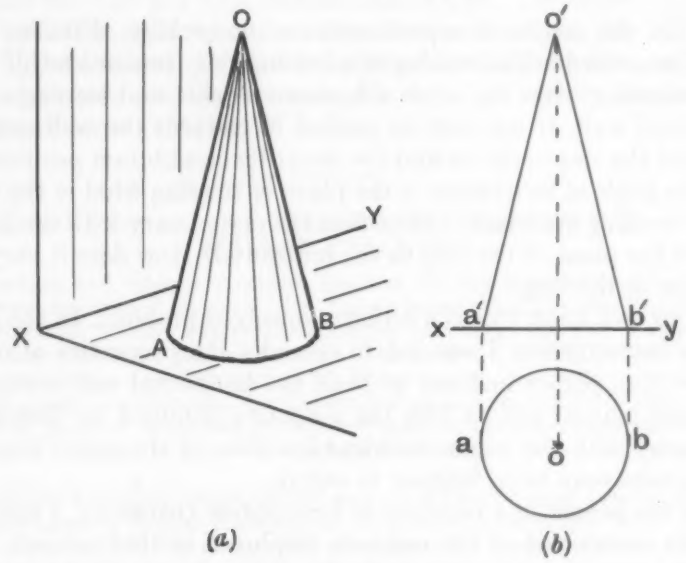

FIG. 1, a and b.

(3) *Oblique Planes:* These are planes which intersect or cut the *HP* and *VP*. The lines of intersection are called the *traces* of the plane. Thus we have a horizontal trace, *HT* and a vertical trace, *VT*. I shall limit myself to two cases only of these planes.

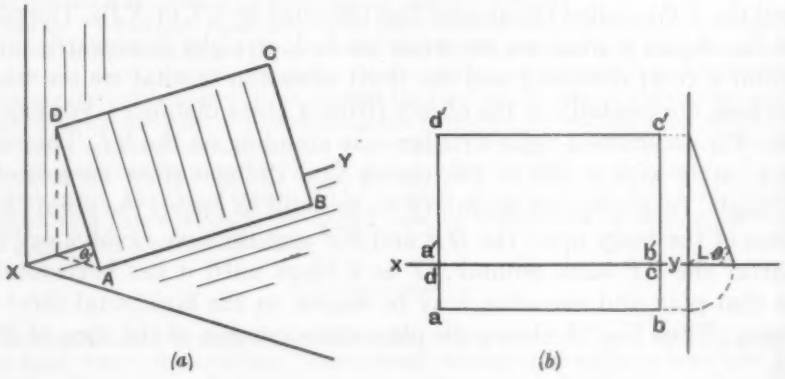

FIG. 2, a and b.

(1) The oblique plane is parallel to XY and cuts the HP and VP in traces which are parallel to XY. Such a plane is seen in perspective as $ABCD$ (Fig. 2a) and in plan and elevation as $abcd$, $a'b'c'd'$ in Fig. 2b. The ring of my rocking experiments lies, when in equilibrium, in such a plane.

Let θ = the inclination of the plane to the HP and L the distance of the plane from XY. Obviously θ and L may be obtained from the drawing. They are also linked by the trigonometrical expressions $\tan\theta = (a'd')/(ad)$ and $L = (ad)\sin\theta$ (see side elevation in Fig. 2b).

(2) The oblique plane cuts the HP and VP; also the ground line XY. Fig. 3a shows a perspective view and Fig. 3b a plan and eleva-

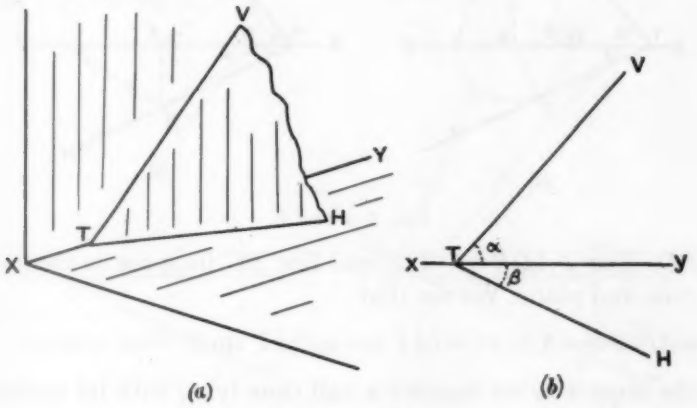

FIG. 3, a and b.

tion. The HT and VT intersect at T. I shall limit myself to cases where $\angle VTH$ of Fig. 3a $< 90°$. Such a plane is one in which my rocking ring lies when it is in a displaced position. The tangents to the ring at the points of contact of the ring with the HP and VP are the HT and VT respectively of Fig. 3b (see also Fig. 12).

(4) Let α, β be the angles between XY (or xy) and VT and HT respectively. Let θ ϕ be the inclinations of the oblique plane to the HP and VP respectively. Let γ be the real angle between VT and HT. Let ψ be the inclination of XY to the oblique plane, i.e. the angle between XY and its projection on the oblique plane.

We now proceed by the help of Descriptive Geometry to find by drawing the relative values of these angles and then to use Trigonometry to connect them.

(4a) *To find θ and ϕ in terms of α and β.* Imagine the half of a vertical cone standing on the HP with its medial section in the VP, its vertex on VT, and of such vertical angle that one of its generating

lines coincides with a line of steepest descent on the oblique plane (Fig. 4a). The inclination of its generating lines to the HP will be θ. To construct this cone take a point A on xy and erect a perpendicular AB cutting VT in B. Draw AC at right angles to HT cutting it in C. With A as center and AC as radius describe a circle cutting xy in D.

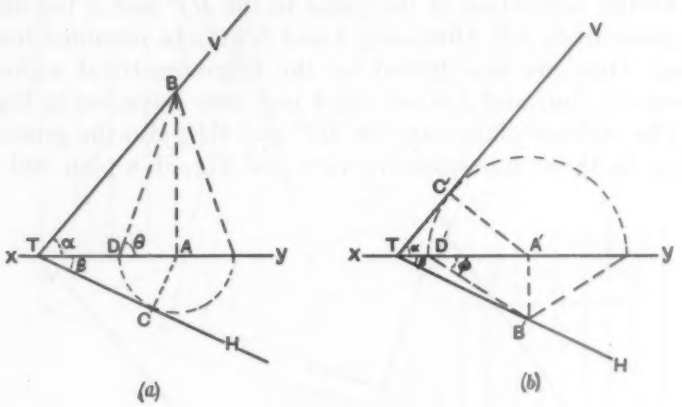

FIG. 4, a and b.

Join BD. The $\angle ADB=\theta$. The real line BC in space is common to both cone and plane. We see that

$$\tan\theta = AB/AD = AB/AC = (AT\tan\alpha)/(AT\sin\beta) = \tan\alpha/\sin\beta. \qquad (1)$$

In the same way we imagine a half cone lying with its medial section in the HP, its vertex on HT and one of its generating lines lying in the given plane. The inclination of this generating line to the VP is ϕ. The construction is similar to that just given for θ and is shown in Fig. 4b using dashed letters as the counterparts of the letters in Fig. 4a. Working as before we find

$$\tan\phi = \tan\beta/\sin\alpha. \qquad (2)$$

(4b) *To find γ in terms of α and β.* Consider a right angled triangle TBC (right angle at C) lying on the plane with TB and TC coincident with the VT and HT respectively (Fig. 5). The $\angle BTC=\gamma$. To construct this triangle erect from any point A on xy a perpendicular AB to cut the VT in B. Draw ACE at right angles to the HT. With T as center and TB as radius describe an arc to cut ACE in E. Join TE. The triangle TEC is thus a rabattement of the real triangle TBC about TC as hinge and the $\angle ETC$ = the real $\angle BTC$ = the real $\angle VTH$ $=\gamma$.

We have $TA=TB\cos\alpha$, $TC=TA\cos\beta=(TB\cos\alpha)\cos\beta$, also $TC=TE\cos\gamma=TB\cos\gamma$.

$$\therefore \quad \cos\gamma = \cos\alpha\cos\beta. \qquad (3)$$

Sin $\gamma/2$ is needed later. By the identity

$$\cos A = 1 - 2\sin^2 A/2$$

we get

$$\sin \gamma/2 = \{\tfrac{1}{2}(1 - \cos\alpha\cos\beta)\}^{1/2}. \qquad (4)$$

(4c) *To find ψ in terms of α and β.* From any point A on Ty (Fig. 6) imagine a perpendicular dropped upon the given plane. If this perpendicular has length L then $\sin\psi = L/TA$. To find the length of the perpendicular imagine that we look at the plane with rays of light parallel to HT, i.e. along the arrow of Fig. 6. The plane is seen in profile at its true slope θ to the HP and the perpendicular L is seen at its true value.

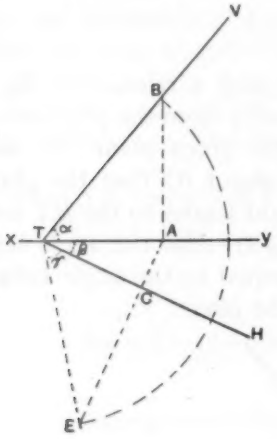

FIG. 5.

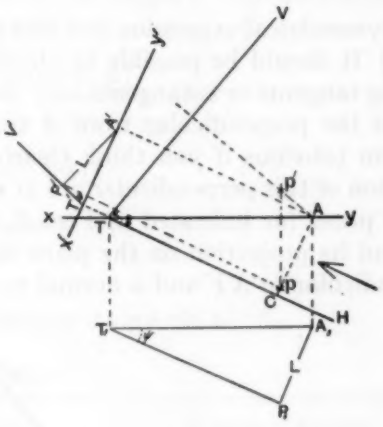

FIG. 6.

Therefore draw a new ground line $x'T'y'$ at right angles to HT produced. Having found θ as in Fig 4 draw $T'V'$ at $\angle\theta$ to $x'y'$. Project A parallel to HT to A' on $x'y'$ and draw $A'P'$ at right angles to $T'V'$. It is the perpendicular required.

We see that its length

$$L = A'P' = A'T' \sin\theta = AC \sin\theta = (TA \sin\beta)\sin\theta.$$

Hence

$$\sin\psi = \sin\beta\sin\theta, \text{ and by symmetry } \sin\psi \text{ also} = \sin\alpha\sin\phi. \qquad (5)$$

The plan and elevation of P the foot of the perpendicular are at p and p' on the lines drawn from A at right angles to the HT and VT respectively. The lower triangle $A_1T_1P_1$ on Fig. 6 shows the true shape of the triangle ATP and the Angle $A_1T_1P_1 = \psi$.

To express ψ in terms of α and β only take the second of equations (5) and eliminate ϕ by the aid of Eq. 2 which is $\tan \phi = \tan \beta / \sin \alpha$. This equation gives

$$\left.\begin{aligned} & \sin \phi = \tan \beta / (\sin^2 \alpha + \tan^2 \beta)^{1/2} \\ \therefore \quad & \sin \psi = \sin \alpha \tan \beta / (\sin^2 \alpha + \tan^2 \beta)^{1/2} \\ \text{and also by symmetry} \quad & \sin \psi = \sin \beta \tan \alpha / (\sin^2 \beta + \tan^2 \alpha)^{1/2} \end{aligned}\right\} \tag{6}$$

Also from Eq. 6

$$\left.\begin{aligned} \tan \psi &= \sin \alpha \tan \beta (\sin^2 \alpha + \tan^2 \beta - \sin^2 \alpha \tan^2 \beta)^{1/2} \\ &= \sin \alpha \tan \beta / (\sin^2 \alpha + \cos^2 \alpha \tan^2 \beta)^{1/2} \\ &= \tan \alpha \tan \beta / (\tan^2 \alpha + \tan^2 \beta)^{1/2} \\ \text{or} \quad \cot^2 \psi &= \cot^2 \alpha = + \cot^2 \beta, \end{aligned}\right\} \tag{7}$$

both symmetrical expression in α and β.

(4d) It should be possible to obtain such a relation as Eq. 7 involving tangents or cotangents only directly from the plan and elevation of the perpendicular from A to the given plane. We use the theorem (obvious if you think clearly about it) that the plan and elevation of this perpendicular are at right angles to the HT and VT of the plane (as indicated in Figs. 5, 6. 7). Also the angle between XY and its projection on the plane is equal to the angle between a perpendicular to XY and a normal to the plane.

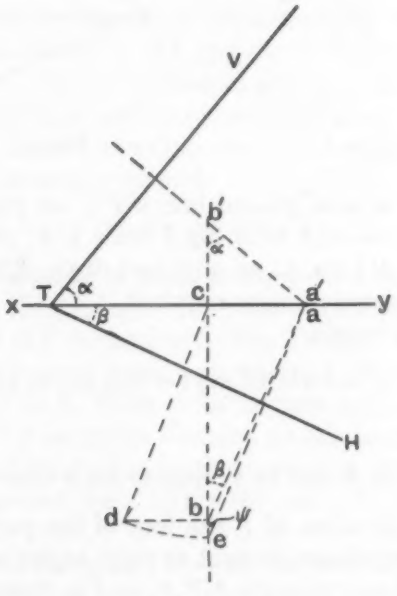

FIG. 7.

Therefore, the construction is—Select a point A (plan a, elevation a') on xy (Fig. 7), draw normals to HT and VT respectively and on these pick a point B whose plan and elevation are b and b' respectively. Let bb' cut xy in c. Draw $bd \perp cb$ and equal to cb'. Then cd is the true length of the real line CB. Rebat the $\triangle CBA$ about ca as hinge, i.e. make $ce = cd$ and join ea, thus getting the $\triangle cea$ and we see that $\angle cea$ is the real angle between perpendiculars to xy and the plane respectively. Hence $\angle cea = \psi$ and

$$\cot^2 \psi = \frac{ce^2}{ca^2} = \frac{cd^2}{ca^2} = \frac{bd^2+bc^2}{ca^2} = \frac{b'c^2+bc^2}{ca^2} = \cot^2 \alpha + \cot^2 \beta. \tag{7}$$

(5) *To express* α, β, γ, ψ *in terms of* θ *and* ϕ. Equation (1) gives $\sin \beta = \tan \alpha / \tan \theta$ and Eq. (2) gives $\tan \beta = \sin \alpha \tan \phi$. By use of the identity, $\sin^2 \beta = \tan^2 \beta / (1+\tan^2 \beta)$ the angle β may be eliminated with the result that we get

$$\left.\begin{aligned} &\cos \alpha = \cos \theta / \sin \phi \\ \text{and by symmetry} \quad &\cos \beta = \cos \phi / \sin \theta \end{aligned}\right\} \tag{8}$$

Eq. (3) is $\cos \gamma = \cos \alpha \cos \beta$, hence by (8)

$$\cos \gamma = \cot \theta \cot \phi \tag{9}$$

and

$$\sin \gamma/2 = \{(-\cos (\theta+\phi))/2 \sin \theta \sin \phi\}^{1/2}. \tag{10}$$

TABLE I

	In terms of α and β	In terms of θ and ϕ
α	α	$\cos \alpha = \cos \theta / \sin \phi$
β	β	$\cos \beta = \cos \phi / \sin \theta$
θ	$\tan \theta = \tan \alpha / \sin \beta$	θ
ϕ	$\tan \phi = \tan \beta / \sin \alpha$	ϕ
γ	$\cos \gamma = \cos \alpha \cos \beta$	$\cos \gamma = \cot \theta \cot \phi$
ψ	$\tan \psi = \dfrac{\tan \alpha \tan \beta}{(\tan^2 \alpha + \tan^2 \beta)^{1/2}}$ or $\cot \psi = (\cot^2 \alpha + \cot^2 \beta)^{1/2}$	$\cos \psi = (\cos^2 \theta + \cos^2 \phi)^{1/2}$

Eq. (5) is $\sin\psi=\sin\alpha\sin\phi$. Use Eq. (8) to substitute for $\sin\alpha$ and we obtain

$$\left.\begin{aligned}\sin\psi&=(\sin^2\theta-\cos^2\phi)^{1/2}=(\sin^2\phi-\cos^2\theta)^{1/2}\\ \text{and}\quad\cos\psi&=(\cos^2\theta+\cos^2\phi)^{1/2}\end{aligned}\right\}. \tag{11}$$

Table I summarizes the connecting relations.

Other connecting relations obtained from the above are:

$$\left.\begin{aligned}\sin\alpha/\sin\beta&=\sin\theta/\sin\phi\\ \cos\alpha/\cos\beta&=\sin 2\theta/\sin 2\phi\\ \tan\alpha/\tan\beta&=\cos\phi/\cos\theta\end{aligned}\right\}. \tag{12}$$

We now deduce a new relation connecting α, β, γ and ψ.

Eq. (3) is $\cos\gamma=\cos\alpha\cos\beta$ hence

$$\begin{aligned}\tan\gamma&=\frac{(1-\cos^2\alpha\cos^2\beta)^{1/2}}{\cos\alpha\cos\beta}=\frac{(1-(1-\sin^2\alpha)\cos^2\beta)^{1/2}}{\cos\alpha\cos\beta}\\ &=\frac{(1-\cos^2\beta+\sin^2\alpha\cos^2\beta)^{1/2}}{\cos\alpha\cos\beta}\\ &=\frac{(\sin^2\beta+\sin^2\alpha\cos^2\beta)^{1/2}}{\cos\alpha\cos\beta}\\ &=\left(\tan^2\alpha+\frac{\tan^2\beta}{\cos^2\alpha}\right)^{1/2}\\ \text{and by symmetry}&=\left(\frac{\tan^2\alpha}{\cos^2\beta}+\tan^2\beta\right)^{1/2}\end{aligned} \tag{13}$$

By Eq. (6)

$$\sin\psi=\frac{\sin\alpha\tan\beta}{(\sin^2\alpha+\tan^2\beta)^{1/2}}.$$

Divide above and below by $\cos\alpha$. Hence

$$\sin\psi=\frac{\tan\alpha\tan\beta}{\left(\tan^2\alpha+\dfrac{\tan^2\beta}{\cos\alpha}\right)^{1/2}}$$

and by the use of Eq. 13 we see

$$\sin\psi=\frac{\tan\alpha\tan\beta}{\tan\gamma}. \tag{14}$$

Eq. 14 may also be obtained from Figs. 4, 5, 6. For

$$\sin\psi = \frac{L}{AT} = \frac{A'P'}{AT} = \frac{AC\sin\theta}{AT} = \frac{AC}{AT}\,\frac{AB}{(AB^2+AC^2)^{1/2}} = \frac{AC}{AT}\cdot\frac{AB}{CE}$$

$$= \frac{AB}{AT}\cdot\frac{AC}{TC}\cdot\frac{TC}{CE} = \left(\frac{AB}{AT}\cdot\frac{AC}{TC}\right)\Big/\left(\frac{CE}{TC}\right)$$

$$= (\tan\alpha\cdot\tan\beta)/\tan\gamma. \qquad (14)$$

(6) *Given θ and ϕ to construct the traces HT, VT and therefore to find the angles α and β.* This geometrical construction is interesting being partly a backward working of the problems illustrated in Fig. 4 and relying for its truth on a certain geometrical fact which is proved below. The construction is: Through A, any point on xy, (Fig. 8) draw a line BAE at right angles to xy. From any point B on BAE draw the line BC making the $\angle BCA = \phi$. Draw AD at right angles to BC. With A as center and radius AD describe the circle DRK. This circle is both plan and elevation of a sphere whose center A is on xy and whose radius $= AD$. Draw a tangent FKE to this circle making the $\angle EFA = \theta$, cutting xy and BAE in F and E respectively and touching the circle in K. With center A and radius AC describe the circle CSM. From E draw a tangent EMT touching this circle in M and cutting xy in T. Join TB, TE. These lines are the horizontal and vertical traces TH, TV of the given plane and the angles α and β are thus found.

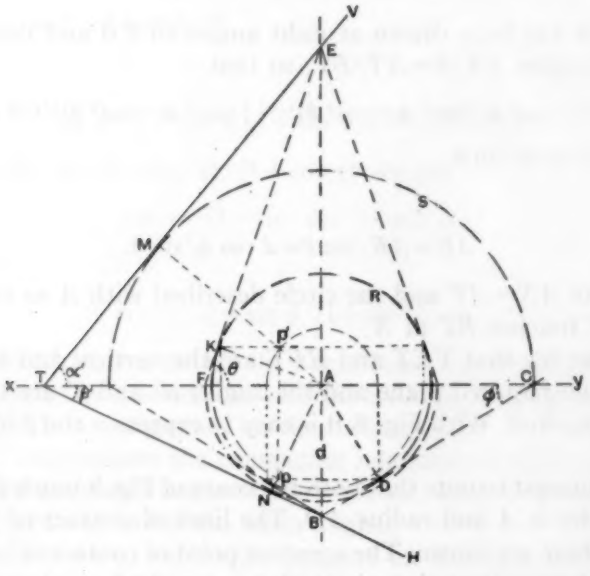

FIG. 8

It is obvious that the construction depends, for its truth, on the fact that AF is equal to the perpendicular AN dropped from A upon TB. This we proceed to prove:

Join AK, AM. Denote the length of AB by d. Until we have proved the truth of the construction denote the angles VTy, HTy by α' and β' respectively. Then by Fig. 8 we see $AM=AC=d\cot\phi$, $AK=AD=d\cos\phi$, $AK=AE\cos\theta$. Therefore $AE=d\cos\phi/\cos\theta$. Now $\cos\alpha'=AM/AE$, therefore

$$\cos\alpha'=(d\cot\phi)/(d\cos\phi/\cos\theta)=\cos\theta/\sin\phi$$

Also

$$AE/AT=\tan\alpha'=(\sec^2\alpha'-1)^{1/2}=(\sin^2\phi/\cos^2\theta-1)^{1/2}$$
$$=(\sin^2\phi-\cos^2\theta)^{1/2}/\cos\theta.$$

Therefore

$$AT=AE/\tan\alpha'=(d\cos\phi/\cos\theta)\cos\theta/(\sin^2\phi-\cos^2\theta)^{1/2}$$
$$=d\cos\phi/(\sin^2\phi-\cos^2\theta)^{1/2}$$

Also

$$BT^2=d^2+AT^2=d^2\{1+\cos^2\phi/(\sin^2\phi-\cos^2\theta)\}$$
$$=d^2(1-\cos^2\theta)/(\sin^2\phi-\cos^2\theta).$$

Therefore

$$BT=d\sin\theta/(\sin^2\phi-\cos^2\theta)^{1/2}.$$

Now AN has been drawn at right angles to TB and therefore by similar triangles $AN/d=AT/BT$, so that

$$AN=d\{d\cos\phi/(\sin^2\phi-\cos^2\theta)^{1/2}\}\{(\sin^2\phi-\cos^2\theta)^{1/2}/d\sin\theta\}$$
$$=d\cos\phi/\sin\theta.$$

also

$$AF=AK/\sin\theta=d\cos\phi/\sin\theta.$$

Therefore $AN=AF$ and the circle described with A as centre and radius AF touches BT at N.

Hence we see that VET and HBT are the vertical and horizontal traces of the required plane and the angles α' and β' are the angles α and β required. With Fig. 8 it is easy to express α and β in terms of θ and ϕ.

It is of interest to note that both the cones of Fig. 8 touch the sphere whose centre is A and radius AD. The lines of contact of the cones and the sphere are shown. The common point of contact of both cones and the sphere is the point whose plan is p and whose elevation is p'.

(7) *A particular case: the symmetrical case.* The author was especially interested in the case where the rocking ring was, when in equilibrium, lying in a plane inclined at 45° to both the *HP* and *VP*. As the ring rocks the oblique planes which contain its positions are always inclined at equal angles to the *HP* and *VP*, i.e. in all positions $\alpha=\beta$ and $\theta=\phi$.

The use of β and ϕ may now be dropped and the connecting equations simplify as follows:

Eq. 1 becomes $\tan\theta=1/\cos\alpha$

$$\tan\theta=1/\cos\alpha.$$

Eq. 3 becomes $\cos\gamma=\cos^2\alpha=\cot^2\theta$

Eq. 4 becomes

$$\sin\frac{\gamma}{2}=\frac{1}{\sqrt{2}}\sin\alpha=\frac{1}{\sqrt{2}}(1-\cot^2\theta)^{1/2}$$

Eq. 7 becomes

$$\tan\psi=\frac{1}{\sqrt{2}}\tan\alpha=\frac{1}{\sqrt{2}}(\tan^2\theta-1)^{1/2}.$$

Eq. 5 becomes

$$\begin{aligned}\sin\psi&=\sin\alpha/(1+\cos^2\alpha)^{1/2}=\{(1-\cos^2\alpha)/(1+\cos^2\alpha)\}^{1/2}\\&=\{(1-\cos\gamma)/(1+\cos\gamma)\}^{1/2}=(2\sin^2\theta-1)^{1/2}\\&=(1-2\cos^2\theta)^{1/2}\end{aligned}$$

whence

$$\cos\psi=\{2\cos\gamma/(1+\cos\gamma)\}^{1/2}=\sqrt{2}\cdot\cos\theta$$

and from $\sin^2\psi=(1-\cos\gamma)/(1+\cos\gamma)$ we get

$$\cos\gamma=(1-\sin^2\psi)/(1+\sin^2\psi)$$

and hence

$$\sin(\gamma/2)=\sin\psi/(1+\sin^2\psi)^{1/2}. \tag{15}$$

Eq. 15 which connects half the angle between the traces with the angle between the ground line and the oblique plane proved useful in the author's investigations on the rocking ring.

Table II summarizes the connecting relations for the symmetrical case. In this case each angle of the four, i.e., α, θ, γ, ψ, may be expressed in terms of one of the other angles.

(8) *The Bisector of an Angle.* Imagine an isosceles triangle lying on an oblique plane. The plan (and elevation) of the triangle will be

an isosceles triangle only if the bisector of the vertical angle of the triangle is a line of steepest descent on the plane, and in this case the plan (and elevation) of the bisector of the vertical angle bisects the plan (and elevation) of the vertical angle.

TABLE II

The Symmetrical Case

In terms of:

	α	θ	γ	ψ
α	α	$\cos\alpha=\cot\theta$	$\cos\alpha=(\cos\gamma)^{1/2}$	$\tan\alpha=\sqrt{2}\cdot\tan\psi$
θ	$\cot\theta=\cos\alpha$	θ	$\cot\theta=(\cos\gamma)^{1/2}$	$\cos\theta=\frac{1}{\sqrt{2}}\cos\gamma$
γ	$\cos\gamma=\cos^2\alpha$	$\cos\gamma=\cot^2\theta$	γ	$\cos\gamma=\cos^2\psi/(1+\sin^2\psi)$
$\gamma/2$	$\sin\frac{\gamma}{2}=\frac{1}{\sqrt{2}}\sin\alpha$	$\sin\frac{\gamma}{2}=\frac{1}{\sqrt{2}}(1-\cot^2\theta)^{1/2}$	$\gamma/2$	$\sin\frac{\gamma}{2}=\sin\psi/(1+\sin^2\psi)^{1/2}$
ψ	$\tan\psi=\frac{1}{\sqrt{2}}\tan\alpha$	$\cos\psi=\sqrt{2}\cdot\cos\theta$	$\sin\psi=\left(\frac{1-\cos\gamma}{1+\cos\gamma}\right)^{1/2}$	ψ

If the bisector of the vertical angle is not a line of steepest descent the two equal sides of the isosceles triangle do not project as equal sides in the plan (and elevation) although the bisector will always bisect the plan (and elevation) of the third side of the triangle. Hence in general the plan (and elevation) of the bisector of the vertical angle of an isosceles triangle lying on an oblique plane will not bisect the plan (and elevation) of the vertical angle.

The author was interested in the bisector of the real angle VTH between the traces of an oblique plane, i.e. the bisector of the angle γ (Fig. 9). How do the plan and elevation of this bisector TF subdivide the angles α and β? Note that in general the bisector TF is not coincident with the projection of the ground line XY on the oblique plane. It is coincident only in the symmetrical case (see §7).

(8a) *First method of dealing with the bisector of the angle* γ. Take

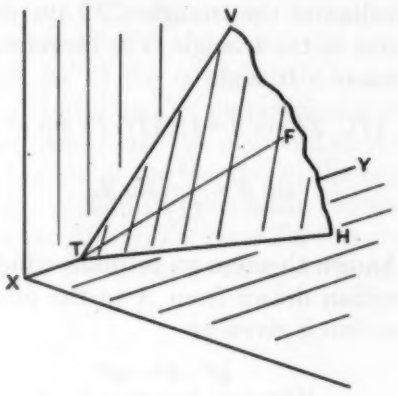

FIG. 9

an isosceles triangle TBC (Fig. 10a) with vertex at T and the equal sides TB, TC lying along the VT and HT respectively. The plan and elevation of the third side BC are bC and Bc' respectively. The bisector of γ bisects BC. Hence bisect bC in f and Bc' in f'. Draw Tf, Tf'. These are the plan and elevation of the bisector TF.

The angles VTy, HTy are α and β (as before). Denote the angles $f'Ty$, fTy by α' and β' respectively.

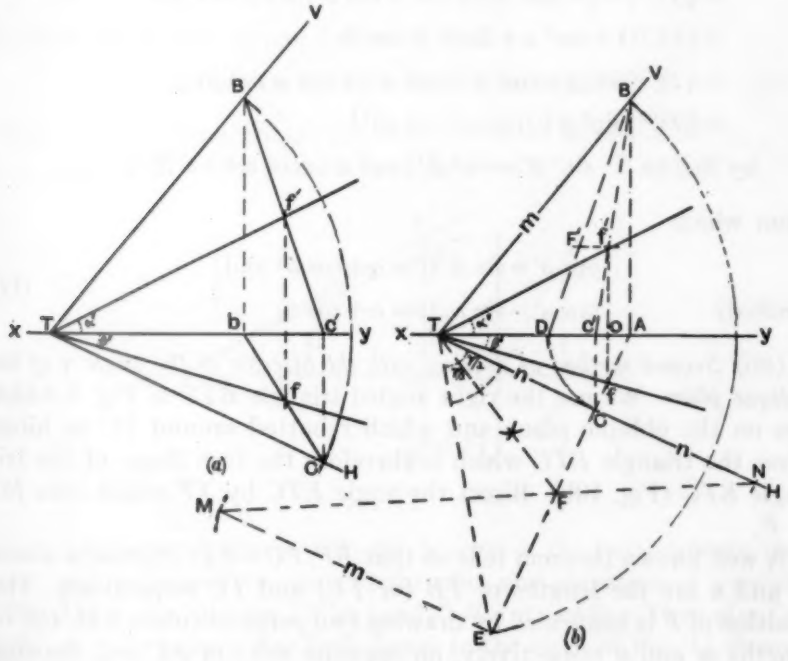

FIG. 10, a and b.

Since Tf is a median of the triangle CTb the area of the triangle CTb is twice the area of the triangle fTb. Therefore by a well known formula for the area of a triangle

$$\tfrac{1}{2}TC \cdot Tb \sin \beta = 2 \times \tfrac{1}{2}Tf \cdot Tb \sin \beta'$$

$$\therefore \quad \sin \beta' = \frac{TC}{2Tf} \sin \beta. \tag{16}$$

There is a well known theorem on medians which says that if in a triangle ABC a median drawn from A to the mid point of BC the length M of this median is given by

$$M^2 = -\frac{a^2}{4} + \frac{b^2}{2} + \frac{c^2}{2}.$$

Apply this theorem to the median Tf of the triangle Tcb. Therefore

$$Tf^2 = -\frac{Cb^2}{4} + \frac{TC^2}{2} + \frac{Tb^2}{2}.$$

Now $Cb^2 = TC^2 + Tb^2 - 2TC \cdot Tb \cos \alpha$, also $Tb = TB \cos \alpha = TC \cos \alpha$.

$$\therefore \quad Tf^2 = -\tfrac{1}{4}(TC^2 + TC^2 \cos^2 \alpha - 2TC^2 \cos \alpha \cos \beta + \tfrac{1}{2}TC^2 + \tfrac{1}{2}TC^2 \cos^2 \alpha$$

$$= \tfrac{1}{4}TC^2(-1 - \cos^2 \alpha + 2 \cos \alpha \cos \beta + 2 + 2 \cos^2 \alpha)$$

$$= \tfrac{1}{4}TC^2(1 + \cos^2 \alpha + 2 \cos \alpha \cos \beta)$$

$$= \tfrac{1}{4}TC^2(\sin^2 \beta + \cos^2 \beta + \cos^2 \alpha + 2 \cos \alpha \cos \beta)$$

$$= \tfrac{1}{4}TC^2\{\sin^2 \beta + (\cos \alpha + \cos \beta)^2\}$$

$$\therefore \quad \text{by Eq. 16} \qquad \sin^2 \beta' = \sin^2 \beta / \{\sin^2 \beta + (\cos \alpha + \cos \beta)^2\}$$

from which

$$\left.\begin{aligned} &\tan \beta' = \sin \beta / (\cos \alpha + \cos \beta) \text{ and} \\ \text{similarly} \quad &\tan \alpha' = \sin \alpha / (\cos \alpha + \cos \beta) \end{aligned}\right\}. \tag{17}$$

(8b) *Second method of dealing with the bisector of the angle γ of the oblique plane.* We use the right angled triangle BTC of Fig. 5 which lies on the oblique plane and which rebatted around TC as hinge gives the triangle ETC which is therefore the true shape of the triangle BTC (Fig. 10b). Bisect the angle ETC by TF which cuts EC in F.

A well known theorem tells us that $EF/FC = ET/TC = m/n$ where m and n are the lengths of TB (or TE) and TC respectively. The position of F is confirmed by drawing two perpendiculars EM, CN of lengths m and n respectively, on opposite sides of EC and drawing MN which cuts EC in F. To find the plan and elevation of F, obtain

AD as in Fig. 4. We see $DB=EC$, so measure up DF' equal to CF. F' gives the height of F above the HP. Bc' and AC are the elevation and plan of the side BT of the triangle BTC so draw $F'f'$ parallel to xy to meet Ac in f' and draw $f'of$ perpendicular to xy cutting xy in o and AC in f. Then f and f' are the plan and elevation of F and Tf, Tf' the plan and elevation of the bisector of the angle γ. Denote the angles ATf', ATf by α' and β'. We must express these in terms of α and β. We have $\tan\beta'=of/To$. Now $TA=m\cos\alpha$, $TC=m\cos\alpha\cos\beta=n$, $AC=TA\sin\beta=m\cos\alpha\sin\beta$, $Ac'=AC\sin\beta=m\cos\alpha\sin^2\beta$,

$$Ao=\frac{m}{m+n}Ac'=\frac{m^2}{m+n}\cos\alpha\sin^2\beta.$$

$$\therefore\quad To=TA-Ao=m\cos\alpha\left(1-\frac{m}{m+n}\sin^2\beta\right)$$

$$=\frac{m}{m+n}\cos\alpha(m\cos^2\beta+n)=\frac{m^2}{m+n}\cos\alpha\cos\beta(\cos\beta+\cos\alpha).$$

Also

$$of=\frac{m}{m+n}Cc'=\frac{m}{m+n}(n\sin\beta)=\frac{m^2}{m+n}\cos\alpha\cos\beta\sin\beta$$

Therefore $\tan\beta'=\sin\beta/(\cos\alpha+\cos\beta)$ and

$$\tan\alpha'=\sin\alpha/(\cos\alpha+\cos\beta)\text{ as before.}\qquad(17)$$

Note. In the symmetrical case where $\alpha=\beta$ we have $\tan\alpha'=1/\sqrt{2}\tan\alpha$ and $\tan\beta'=1/\sqrt{2}\tan\beta$ and therefore $\alpha'>a/2$ and $\beta'>\beta/2$.

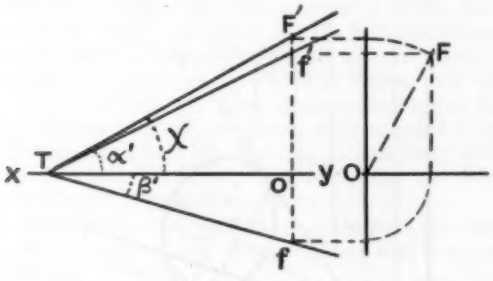

FIG. 11

(8c) *The angle χ between TF, the bisector of the angle between the traces, and the ground line xy.* Let the plan and elevation of a point F (Fig. 11) on the bisector TF be f and f'. ff' cuts xy in o. Then $of=To\tan\beta'$, $of'=To\tan\alpha'$. The little figure to the right of Fig. 11 shows F and O in a side elevation. Obviously $OF=To(\tan^2\alpha'+\tan^2\beta')^{1/2}$

and $OF/TO=\tan\chi$.

$$\therefore \quad \tan\chi=(\tan^2\alpha'+\tan^2\beta')^{1/2}$$

which by Eq. 17 becomes

$$\tan\chi=(\sin^2\alpha+\sin^2\beta)^{1/2}/(\cos\alpha+\cos\beta). \tag{18}$$

To get χ geometrically revolve OF around O as center to a vertical position, project back to cut fof' in F'. Join TF'. The $\angle F'Ty=\chi$.

In the symmetrical case where $\alpha=\beta$, $\tan\chi=(1/\sqrt{2})\tan\alpha$ and χ becomes ψ thus agreeing with $\psi=(1/\sqrt{2})\tan\alpha$ of §7.

Eq. 18 gives χ in terms of α and β. To express it in terms of θ and ϕ make the proper substitutions (Eq. 8) and so get

$$\tan\chi=\frac{(2\sin^2\theta\sin^2\phi-\sin^2\theta\cos^2\theta-\sin^2\phi\cos^2\phi)^{1/2}}{(\sin\theta\cos\theta+\sin\phi\cos\phi)} \cdots \tag{19}$$

which for the symmetrical case reduces to

$$\tan\chi=\frac{1}{\sqrt{2}}(\tan^2\theta-1)^{1/2}, \text{ as in §7.}$$

(9) *The problem of the rocking ring.* When the leaning ring is at rest in its equilibrium position it may be said to be in an oblique plane such as is shown in Fig. 2a. When the ring is in a displaced position it may be said to be in an oblique plane such as is shown in Fig. 3a. In the equilibrium position the ring touches the VP and the HP at P and Q on the ring; PQ is the diameter of steepest slope. In all positions of the ring the VP and HP are tangential to the ring and in displaced positions such as shown in Fig. 12 the tangents at R and S, the instantaneous points of contact, meet at T on the ground line

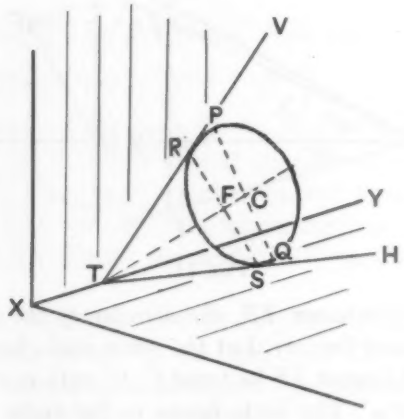

FIG. 12

XY. These tangents are the HT and VT respectively of the oblique plane. As the ring oscillates this plane oscillates, T moving to and fro above XY. If C is the center of the ring the line TC bisects the angle RTS, but only in the case where the oblique plane always makes equal angles with the HP and VP is the plane TCY perpendicular to the plane of the ring.

In all displaced positions the center of the ring is higher than in the equilibrium position so that the potential energy of the ring is greater in the displaced positions, and during the rocking there is a continual interchange between the potential energy and the kinetic energy as in the case of an ordinary pendulum. For any one consistent set of the variables $\alpha, \beta, \theta, \phi, \gamma$ and ψ it should be possible to calculate the value of this increase of potential energy and then proceed by the usual processes of mechanics to find the period of rocking. For the general position this task is not easy. It is not too hard, however, for what we have called the symmetrical case, i.e. the one in which the equilibrium position of the plane is equally inclined to the HP and the VP and in this case we find the period

$$T = 2\pi\sqrt{\frac{R/\sqrt{2}}{g}}$$

where R is the radius of the ring and g the acceleration of gravity.

SWEDISH SCIENTISTS MAKE ELEMENT 100

Swedish scientists have manufactured the heaviest element in the universe, number 100.

This is believed to be the first time that scientists outside the United States have succeeded in making an element heavier than uranium, number 92, which is the heaviest occurring naturally. Announcement of the discovery of element 100 by a University of California group led by Nobelist Glenn T. Seaborg was made last February, but the date of their work was not given.

The Swedish scientists report in the *Physical Review* that their "first positive results were obtained" on Feb. 19. They made an isotope of the element 100 by bombarding the A-bomb element, uranium, with charged oxygen particles in a cyclotron. The isotope has a half life of about half an hour, Drs. Hugo Atterling, Wilhelm Forsling, Lennart W. Holm, Lars Melander and Bjorn Astrom of the Nobel Institute of Physics, Stockholm, report.

Use of oxygen as a bombarding particle in cyclotrons is a recent process. Other heavy particles, such as carbon and nitrogen, have also been used in place of hydrogen and helium.

The Swedish scientists made the element 100 with oxygen ions having energies of 180,000,000 electron volts. The uranium was forced to take on, in one step, the eight protons necessary to change it into the new element, number 100.

Packless pump, hermetically sealed and completely sanitary, is designed for use in medical, food processing, chemical and oil industries. The pump, which works on a vibration principle, needs no lubrication and can be taken apart and cleaned in a few minutes.

WHAT IS MASS?

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In the light of modern atomic knowledge, it would seem that the definition of mass, as still given in reference books, and in at least some text books, should be changed.

The short definition of mass, as found in the places mentioned, is, "The quantity of matter in a body," or something similar. It is additionally explained, of course, that mass is the same everywhere, while weight, which is the measure of gravitational pull, varies from earth to moon, planet to planet, and even at different locations on the earth, since objects are slightly heavier at the poles than at the equator. Thus, a man, who weighs 180 pounds on the earth, would weigh only 30 pounds on the moon, but his mass would remain the same in both locations.

The old definition of mass was possibly excusable in preatomic times, but now it compares with the scientific facts about as the flintlock musket compares with the atomic bomb.

Scientists now are aware that the mass of any object is wholly dependent upon the kind and number of atoms it contains. Therefore, it is suggested that the short definition of mass should be: "Mass is atomic content."

This definition satisfies every requirement, and is specific. Identical numbers of atoms of the same isotope of an element will always have the same mass. This will be true, even though some of the atoms are in the form of a gas, others a liquid, and others a solid. Identical numbers of the atoms of the different forms will weigh the same at the same place.

Masses of quantities of different elements can be compared by finding the number of atoms in each, then calculating how many atoms of the lighter element are required to equal one atom of the heavier element. In this calculation, to be exact, the comparative masses of the varying numbers of electrons must be included. That is, when comparing Uranium 238 and Carbon 12 for instance, we must consider not only the nuclei counts—238 and 12—but also the slight masses of the 92 electrons of the uranium atom and the 6 electrons of the carbon atom.

Equivalent masses, as thus calculated, will weigh the same at the same place.

Masses of compounds can also be measured by atomic count of each element included, followed by comparative calculations.

From all this it would appear that "quantity of matter" as a definition of mass, should be consigned to oblivion.

OPTICAL ILLUSIONS: A PRESENTATION FOR HIGH SCHOOL MATHEMATICS STUDENTS

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The topic of optical illusions should be of special interest to high school students of mathematics. For those that have not been previously introduced to the topic, it should provide for a new awareness to the need for accurate measurement devices.

From a discussion of various optical illusions may come generalizations that are applicable to persons, places, and things. The interest generated may provide students an opportunity to become familiar with line segments, concentric circles, parallel lines, et cetera, in some new and unusual relationships.

Few writers have attempted to classify optical illusions as has been done in the following presentation. This may provide somewhat of a challenge to the reader for improvement. Regardless, the material should indicate to some degree how a topic for the mathematics classroom can be both educational and entertaining.

AN INTRODUCTION TO OPTICAL ILLUSIONS

It has been said that "people should believe nothing they hear and only half of what they see." Most of us will readily admit that facts become greatly distorted when passed by word of mouth, but few are willing to believe that their own eyes will deceive them.

Among the records of our courts of law there are numerous cases in which two or more persons have given conflicting reports of a single incident that they had witnessed; yet there is every indication that they reported truthfully. Such conflicting reports are not only possible, but in some instances very probable. It may be that there are a number of interpretations of what is seen, or that different impressions are obtained when viewed from different positions. Illustrations of how our eyes may deceive us are given on this and the following pages.

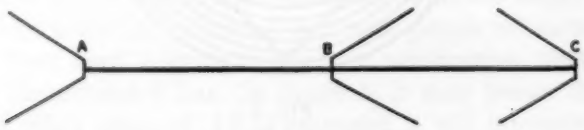

FIG. 1

In figure 1, the segment AB may appear to be longer than the segment *BC*. Both segments, however, are the same length. The addi-

tional oblique segments about the segments AB and BC give the impression of lengthening and shortening, respectively.

In figure 2, the height of the hat (AB) may appear to be of greater length than the width of the brim (CD). Actually the width of the brim of the hat is of greater length than is its height. This may be hard to believe until substantiated by measuring instruments.

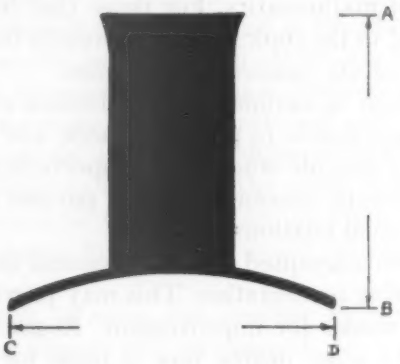

FIG. 2

In figure 3, a square placed among concentric circles appears distorted. Until the sides are lined up along a straight edge, it may be difficult to believe that they are really straight line segments.

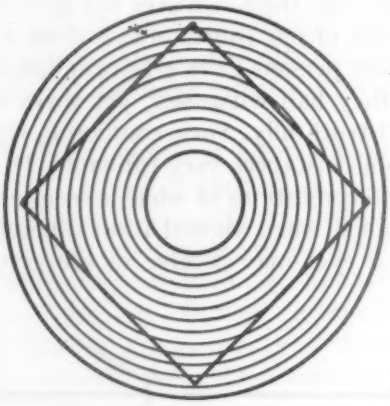

FIG. 3

The figures thus far observed provide examples of optical illustions. There are many different types of optical illusions, a few of which will be further illustrated and discussed.

Some Different Types of Optical Illusions

The height-width illusion. This illusion is quite common among our physical surroundings and frequently causes errors in estimating lengths. An upright segment usually appears to be longer than an equal horizontal segment. For example, in figure 4, the height of the monument seems of greater dimension than the heavy line at its base. Actually the lengths of the heavy base line and the height of the monument are the same. Architects and designers must be on guard for this illusion. We should all be concerned about it in choosing styles of clothes, or in arranging the furnishing in a home to provide the greatest warmth and beauty. When purchasing canned goods it pays to observe the contents of a can, rather than its appearance, as good cans are sometimes designed to provide the illusion of greater size.

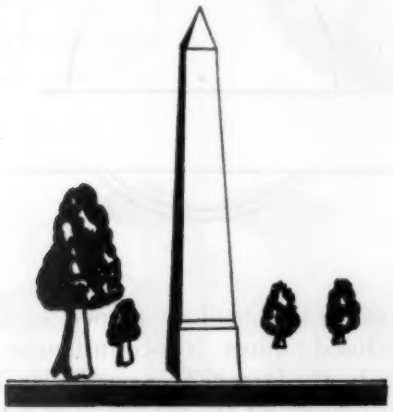

Fig. 4

The illusion of interrupted extent. In this illusion the distance between portions of an interrupted figure seem to shrink, giving the impression that its parts are out of position. In figure 5, for example, one of the three segments on the left side of the pair of parallel lines is extended on their right. At first glance the extended line appears to be the lower segment at the left. A less conservative person might feel that the middle line is the one extended, but most people refuse to believe, until they have used a straight edge for verification, that the top segment is the extended line. In figure 6, it may prove difficult to believe that when segment AB is extended it will intersect at right angles with CD at point C. Similarly, figure 7 might lead one to wonder which arc in the lower part of the diagram goes with the interrupted circle in the upper portion. Believe it or not, it is the innermost arc.

FIG. 5.

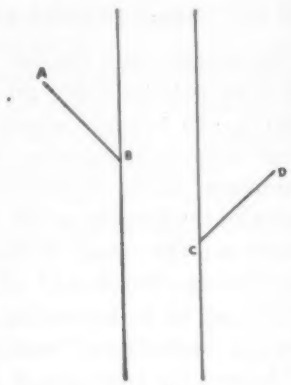

FIG. 6.

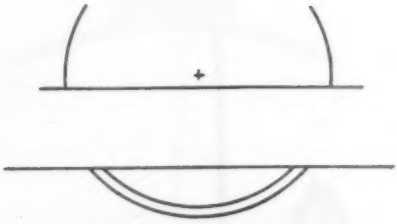

FIG. 7

The illusion of contour. In this illusion, open figures appear to have greater areas than closed figures. Notice in figures 8 and 9 that the open figures seem to be the larger. This particular illusion may cause misjudgement when comparing the capacities of open and closed con-

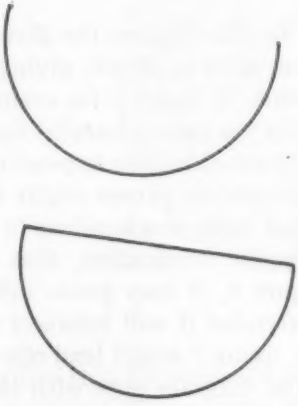

FIG. 8

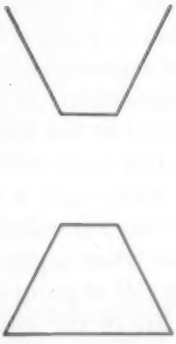

FIG. 9

tainers. A vessel that is covered may appear to be of smaller size than a duplicate that is uncovered.

The equivocal illusion. A figure or diagram that has two or more interpretations is sometimes referred to as an equivocal illusion. Such illusions quite often occur in attempting to illustrate a third dimension on a plane surface. Just when you are certain you have an illustrated object fixed in your mind it will suddenly appear to change its shape. For example, the diagram in figure 10 may appear to be a block

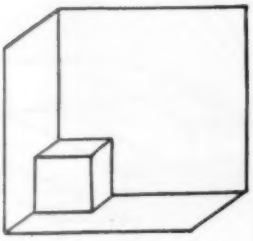

FIG. 10

FIG. 11

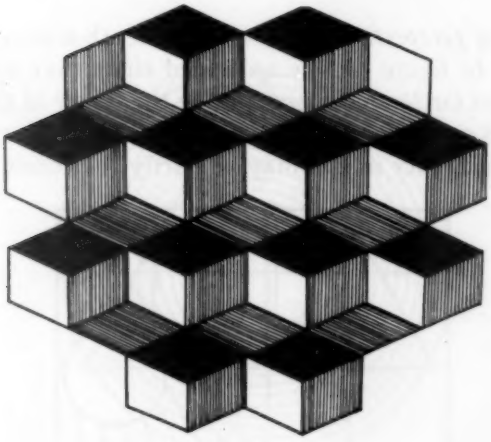

FIG. 12

in front of a block, a block in a corner, or a small block out of a corner of a large block. Figure 11 may appear as a six pointed star, six diamonds, or three blocks within a circle. Figure 12 offers a challenge to determine the number of blocks illustrated. Some persons count twelve blocks, some count nine, while others can't make up their minds how many are included in the figure. Other equivocal illusions are illustrated in figure 13.

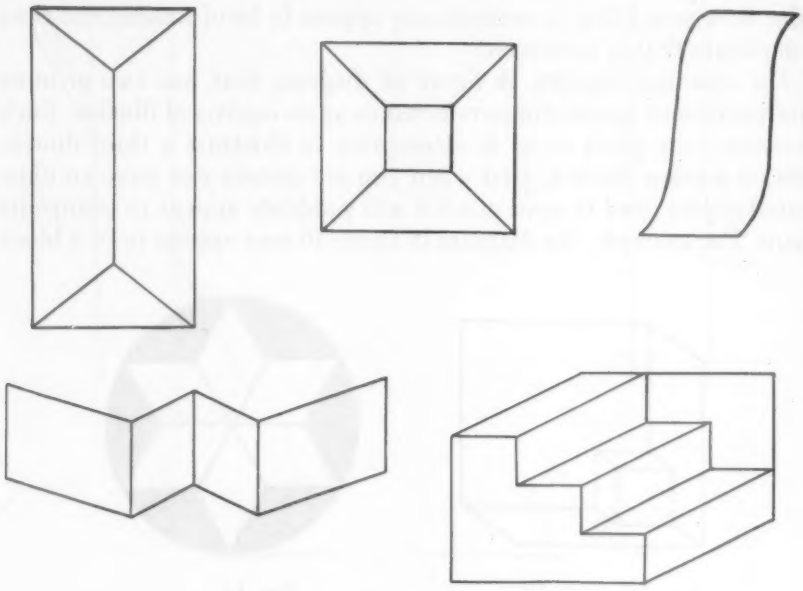

FIG. 13

The illusion of perspective. The illusion of depth makes equal objects seem unequal. In figure 14, the unshaded circle may seem to be beyond the shaded circle and appears to be the larger of the two. Actually both circles are the same size. In addition to the illusion of depth, the apparent difference in size may be partly the result of a light ob-

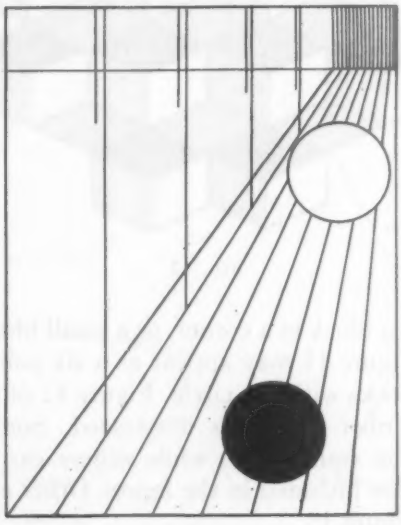

FIG. 14

ject appearing larger than a dark object of equal size. Such illusions are of considerable significance to designers, artists, and engineers. Figure 14 illustrates the problem presented to an officer on the bridge of a ship when he is faced with determining the distance between his ship and another ship. He wonders if the other ship is small and close upon him, or is large and a considerable distance away. It is very difficult to estimate distance without special measuring instruments unless other objects of known size are nearby with which to make comparisons.

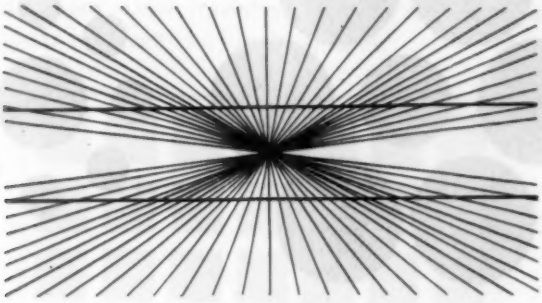

FIG. 15

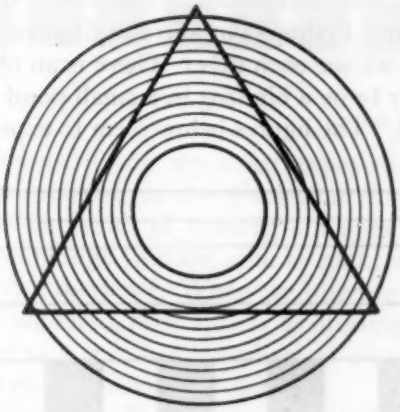

FIG. 16

Illusion of contrast. In this illusion the position of other lines or objects may cause figures to appear distorted. Equals seem unequal; straight lines appear to curve. Can you believe that the two "heavy" lines in figure 15 are parallel segments? This illusion caused the ancients considerable concern in their design of columns used to decorate or support large buildings. They found that a cylindrical column

appeared to bulge in the middle. This was remedied by making the ends of the column of greater diameter than its middle.

Notice in figure 16 how an equilaterial triangle placed among a group of concentric circles appears distorted.

In figure 17, the center circles in each group of circles are equal, though one may appear to be considerably larger than the other. This

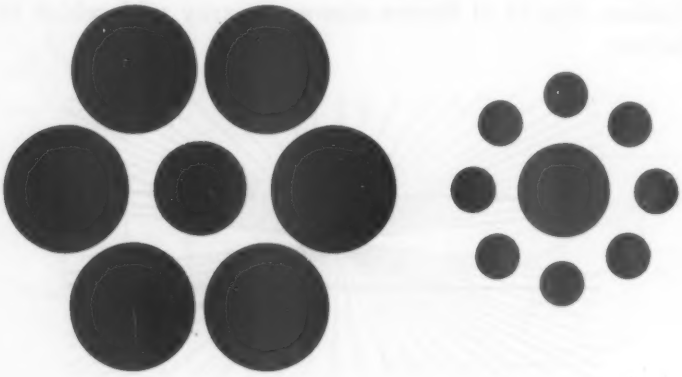

FIG. 17

illusion illustrates how surroundings can completely change the appearance of a figure. Perhaps our surroundings can similarily change the way in which we see each other. There is an old adage that indicates, "It is better to be a big frog in a small pond than to be a small frog in a big pond." The figures below seem to express this adage.

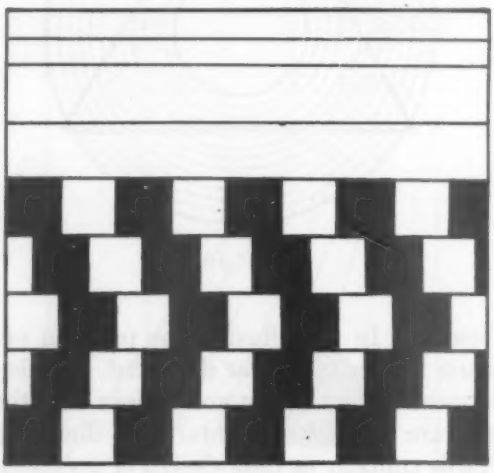

FIG. 18

Contrasting colors often produce interesting illusions. Color contrasts are frequently used by the military to camouflage the shape and position of possible targets from the enemy. When observed from a distance it is exceedingly difficult to tell the direction in which a well camouflaged ship or tank is traveling. Figure 18 illustrates how contrasting colors can provide the illusion of distortion used for camouflage. A pair of rulers placed along the lines in the figure may be necessary to convince you that they are really parallel.

Should you see spots before your eyes in figure 19, it doesn't mean that you need glasses. Gaze intently at the figure and you should see dark spots appear between the black squares.

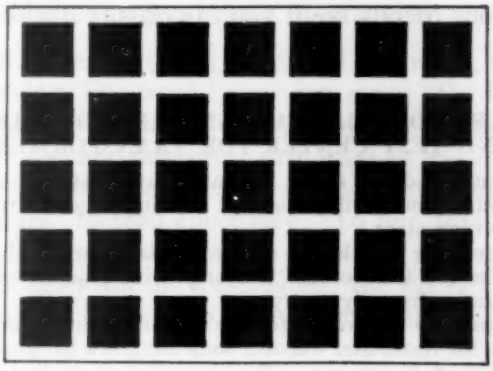

FIG. 19

Other optical illusions. There are other types of optical illusions that have not been included in the discussion thus far. Nearly all optical illusions are the result of training our eyes to adjust to the pattern of our surroundings. We are able to make observations and comparisons only by experience. When we observe things to which our senses have not adjusted, they appear as illusions. Some illusions, however, may occur as a result of the physical characteristics of the eyes. Examples of two such illusions follow:

How to see through a solid object. If you wish to try an experiment, here is a method by which you can apparently see through a solid object. All that is required is a tube of paper about an inch in diameter and about a foot long. Such a tube can be made by rolling a sheet of paper around a finger. Look through the tube with the right eye while in the other hand hold an object, such as a matchbox, alongside of the tube and in front of the left eye. Keep both eyes open and it will appear as though you are looking through a hole in a solid object.

The house that wasn't there. Another interesting experiment involves the lasting impressions that are made on the retina of the eye. Obtain

a sheet of bright red paper and two pieces of white paper of similar size. On the sheet of red paper draw a picture of a house. Cut out the drawing of the house and mount it on one of the pieces of white paper Hold this sheet of paper in one hand and the second white sheet of paper in the other. Gaze steadily at the paper having the design for about half a minute, then immediately transfer your gaze to the blank sheet of paper. You will see the drawing appear there in its complementary color. Drawings can be made in similar fashion of other objects.

Now that you have been introduced to some of the various optical illusions, see if you can construct illusions of your own. After you have determined that you have really produced an optical illusion by trying it on your friends, see if you can classify it according to type.

WESTINGHOUSE TO BUILD MULTI-MILLION-DOLLAR SOUND LAB AND TEST CENTER FOR TRANSFORMERS

A new multi-million-dollar sound laboratory and test center for the world's most powerful transformers will be built at Westinghouse Electric Corporation's Transformer Division plant.

Frank L. Snyder, vice president in charge of this major Westinghouse division, gave details of the new transformer "indoor proving ground" which he said will be ready for operation by early next year.

To be erected in a 200-foot-long extension of four existing assembly aisles at the big Sharon plant, the test center will offer something new in industrial operations, Mr. Snyder explained. Every one of the big power giants that is manufactured will be carried by overhead crane into the test center and undergo a series of searching examinations and electrical blows before it is shipped to the customer.

At one point, the transformer can be bombarded by 5,000,000 volts and 275,000 amperes worth of man-made lightning. Only a few feet away from this thunderous noise will be one of the quietest spots in the world—a building erected inside the over-all test building and housing an ultra-modern sound laboratory.

"This sound laboratory will be 70 feet long, 56 feet wide and 55 feet high," Mr. Snyder said. "It will be constructed so that it will be virtually impossible for sound to enter or to leave the laboratory. The floor will be concrete three feet thick. The five-foot-thick walls will be made of concrete, several inches of air space, and Fiberglas. On the inside, the walls will be covered with Fiberglas wedges and wire mesh—egg crate style—to prevent reflection of sound.

"Within the walls of the sound laboratory will be metallic shielding to prevent even radio waves from entering the lab and interfering with tests.

"A well insulated sliding door of steel and concrete will separate the sound lab from the outside world—and from the indoor test center itself."

The soundproof room will be large enough to test transformers weighing up to 400 tons and rated at greater than 500,000 kilovolt-amperes—much more powerful and larger than any transformers now being manufactured, Mr. Snyder pointed out. These sound tests assure that the powerful transformers will operate as silently as possible.

Hand truck has a hydraulic lift, similar in purpose to those on factory fork-lift trucks. One man can handle 500-pound boxes. The lift operates like a manual auto jack, raising loads as high as three feet.

READING, A CONTRIBUTING AND CONCOMITANT FACTOR IN THE STUDY OF SCIENCE

HOMER L. J. CARTER
Director, Psycho-Educational Clinic, Western Michigan College, Kalamazoo, Mich.

In the study of science, the student reads to secure meaning and he also makes use of meaning in order to read more effectively. Meaning which is the product of certain physical and psychological factors can be expressed by the formula:

$$\text{Meaning} = \text{Sensation} \times \text{Mental Content}$$

Obviously, if either sensation or mental content is missing, there can be no meaning. The word "sine" can mean "the function of an angle," "without," or nothing, depending upon the mental content of the reader. One sees and interprets with what he has seen and experienced. Surely, one reads to secure meaning and one uses the summation of "meanings" to read effectively. "To him that hath shall be given." Let us illustrate this concept.

Several years ago while in Washington, the writer found it necessary to wait several hours for a train. In the interim he visited the Corcoran Gallery of Art. In one of the rooms he observed a man studying carefully a picture by an American artist. This painting, although part of a special display, had little meaning to the writer. Two hours later, after hurrying through the entire exhibit, the writer discovered, to his surprise, the same person examining the same picture. Upon inquiry, it was found that this individual was a well-known connoisseur of American art. This critic had mental content with which to interpret the picture and, consequently, the work of art had meaning to the connoisseur but very little, if any, to the uninformed. Amounts of mental content determine degrees of meaning. This fact was again emphasized by the inscription on the facade of the Union Station in Washington. These words are:

He that Would Bring Home the
Wealth of the Indies Must Carry
the Wealth of the Indies with Him
So It Is in Travelling—A Man
Must Carry Knowledge with Him
if He Would Bring Home Knowledge

Obviously, meaning resulting from reading in any subject matter area requires background and mental content and these in turn are built up by reading and experience. Reading is both a contributing and concomitant factor in the study of science.

Developing Mental Content and Ability to Read in the Teaching of Elementary Science

In an elementary science class, it is the *aim* of the teacher to show causal factors affecting rainfall. The *materials* available are texts in elementary science, glass tubing, mercury, thermometers, Bunsen burner, a large dripping pan and other equipment generally found in an elementary science laboratory. The *procedures* employed by the teacher in using these materials to accomplish the specific goal may be briefly summarized as follows.

1. Ask the students to determine from the rainfall maps in their texts the amount of precipitation on the western slopes of the Rocky Mountains.
2. Determine the amount of rainfall on the eastern slopes.
3. Stimulate interest in determining why the marked difference between the amount of precipitation on the western and eastern slopes of the mountains.
4. Attack the following minor problems in solving this major problem.
   a. Why are the prevailing winds moisture-laden?
   b. What are the causes of ocean currents?
   c. Why do winds blow?
   d. Why are the prevailing winds from the west?
   e. Why does condensation take place?
5. Guide the students in a solution of these problems by the following means:
   a. Constant referral of the students to the various parts of their texts such as index, table of contents and the various maps, charts and illustrations made available by the author. Encourage students to ask questions and read for answers.
   b. Construction of a barometer and the illustration of its use.
   c. Demonstration of ocean currents by making use of water in a large dripping pan covered sparsely with sawdust and with one corner placed over Bunsen burner.
   d. Demonstration of condensation on drinking glass filled with ice water.
   e. Development of such terms as high and low pressure areas, air mass, altitude, condensation, convection, dew points, evaporation, Fahrenheit, front, isobars, isotherms, barometric pressure, water vapor, relative humudity, temperature, and velocity.
   f. Demonstration of how to gather *facts* and how to use these facts in establishing *inferences*. Encourage students to substantiate their inferences by careful reading of their texts and supplementary references. A critical evaluation of all inferences should be stressed.

In the activities previously outlined, it is evident that mental content and ability to read can be developed simultaneously. The young student has been encouraged to add words to his vocabulary and to use his text and supplementary materials as he has participated in problem solving activities. He has read to learn and he has learned to read.

Three Reading Skills Essential to the Study of Science

The student working in the scientific field should know how to add words to his vocabulary, he should be able to read a chapter effectively, and he should know how to read for details preparatory to the solution of a problem. He appreciates a knowledge of definite proce-

dures which will help him to acquire these skills. Every teacher should be responsible for their development. Steps for the attainment of these minimal requirements are briefly outlined.

I. *Developing Reading and Spelling Vocabularies*

Reading and spelling vocabularies can be developed if the student is taught to:

A. Select from his textbook a word which he wishes to add to his vocabulary.
B. Locate the word in the dictionary and find the meaning which best fits the context from which it was chosen.
C. Look at the beginning and ending of the word. Pay special attention to phonograms, or small words, within it.
D. Pronounce the word aloud. Be sure to associate each syllable with its corresponding sound and be certain that the proper sequence of syllables is maintained.
E. Spell the word aloud, paying careful attention to each syllable.
F. Trace the word with his index finger.
G. Write the word.
H. Compare the word written with the word that he has selected for study.
I. Repeat this process until he can spell and write the word correctly.
J. Make use of the word in a complete sentence so that its full meaning is adequately expressed.
K. Keep a list of the words he has learned in this way. Try to add at least three new words to his vocabulary each day.[1]

II. *Reading A Chapter Effectively and Well*

Chapter reading can be done effectively if the student is taught to:

A. Read the chapter headings, the introduction and conclusion.
B. Convert each heading into a question and write the question on a 3″×5″ card. For example, the heading "Weather Defined" can be changed to read "What is weather?"
C. Find the answer to each question by reading the material under each heading. Express this answer in his own words on the 3″×5″ card containing the question.
D. Place the cards in a pile, cover the answer written on the top card, read the question, and recite the answer to himself. Repeat this procedure for each question. The cards should be sorted into two piles, one for the questions which can be answered and another pile for questions which cannot be answered. Continue his study until he is certain of the answers to all questions.
E. Shuffle the cards and answer the questions in the new order in which they appear.[2]

III. *Reading for Details in Problem Solving*

To develop skill in problem-solving, especially in such areas as mathematics, chemistry and physics, the student should ask himself the following questions:

A. What is to be found?
B. What facts are known?
C. What other facts are needed that are not included in the problem?
D. What are the steps to go through for a solution? The best way to see the steps is to rewrite the problem, omitting all numbers, so as to get the meaning. The numbers are not important as far as the steps of the interpretation are concerned. Another set of numbers would do just as well. Read the problem as one would a story—for its meaning.

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[1] Carter, Homer L. J. and McGinnis, Dorothy J., *Building a Successful College Career*, Assignment 8. Dubuque, Iowa: Wm. C. Brown Company, 1950.

[2] Carter, Homer L. J. and McGinnis, Dorothy J., *Building a Successful College Career*, Assignment 7. Dubuque, Iowa: Wm. C. Brown Company, 1950.

Some other questions are:

E. Can the student make a picture of the problem or summarize the procedure he should follow in solving the problem?
F. Can he show the actual procedure—the figures he has used?
G. Is the answer reasonable? How can it be proved?[3]

In concluding this discussion, it is well to repeat that in reading one is primarily concerned with meaning which is the resultant of both physical and psychological factors. Furthermore, it has been suggested that mental content, a psychological factor, not only determines meaning but is accumulative in nature. In the study of science then, reading is both a contributing and concomitant factor. As one reads to learn, he learns to read. Consequently, word study, chapter reading, and reading for detail preparatory to problem-solving, are activities essential to study in any subject matter field. As these skills are used, they are further developed. The teacher of science is concerned with both the contributing and concomitant aspects of reading in the learning process.

[3] Carter, Homer L. J. and McGinnis, Dorothy J., *Reading Manual and Workbook*, Assignment XV. New York: Prentice-Hall, Inc., 1949.

A SPONSORED RESEARCH PROJECT AT ADELPHI

Columbia Ribbon and Carbon Manufacturing Co., Inc., Glen Cove, N. Y., has signed Adelphi College, Garden City, N. Y., to a $5,000 research contract.

This marks the first time Adelphi has ever undertaken a "sponsored research" project. The money will support research in the field of duplication processes, centering specifically on the problem of permanency in spirit hectographing. The ultimate aim of the project is to produce a hectograph ink which will not fade over a period of time as do those presently in use.

"Such a discovery will perfect the hectographing techinque, insuring fade-proof office records by means of handy, inexpensive duplication. It will greatly increase the already numerous uses of the widely adaptable spirit hectograph duplicator," according to F. R. Nichols of Manhasset, president of Columbia Ribbon.

Directors of the project are: Douglas A. Newman of Glen Cove, in charge of research for Columbia Ribbon, and Dr. Alfred Vogel of Wantagh, chairman of the Chemistry department of Adelphi College.

INDIANA AND SOUTH CAROLINA STUDY DELINQUENCY

The Indiana Council for Children and Youth is moving ahead with plans for holding 17 regional meetings throughout the State, under the sponsorship of area groups, to discuss the delinquency problem and recommend needed legislation and ways of coordinating local resources. Information about the Indiana plan of meetings can be obtained from Miss Mildred French, Executive Secretary, Indiana Council for Children and Youth, 1330 West Michigan St., Indianapolis 7, Indiana.

A State-wide delinquency conference has been announced in South Carolina. The South Carolina Citizens Committee on Children and Youth is sponsoring the meeting, to be held in Columbia, November 18, 1954. Information about plans for this type of meeting can be obtained from Miss Adele J. Minahan, S.C. Conference of Social Work Office, 1930 College St., Columbia, S. C.

THE CORRELATION OF SCIENCE AND MATHEMATICS IN THE JUNIOR HIGH SCHOOL

LA VERNE S. POWERS
Whittier School, Waukegan, Ill.

This subject is much like the weather, there is much said about it but not much done. However, while the average seventh and eighth grade teacher probably feels she is doing very little in the field of correlation of these two subjects, as a matter of fact she may be doing more than she thinks.

First, we need to ask ourselves how can science and math in general contribute to the growth and development of girls and boys as they go through our elementary school? There are literally hundreds of ways in which this question might be answered. But generally it could possibly be stated: to help children gain the ideals, understandings and skills essential to becoming good citizens; or to narrow it down to the individual, the development of a thinker who is open-minded, fair in judgment, accurate, and free of prejudice and superstition.

Each day in our science and math classes occasions should be provided to identify and understand social procedures and problems. Children need chances to use their minds to suggest ways of solving problems and carrying out these suggestions through working together. They also need the opportunity to check the results—to say to each other, "How well did we do this?" "Was our plan good?" "What would have made it better?"

Above all, our science and math classes should be places that are conducive to physical and mental health of children. Our classroom should be a happy place. It should be a place where children are not afraid to ask questions and say what is on their minds.

Now we might ask ourselves to what extent science and math should be correlated. This should not be done to the extent that the fundamental principles of each field are sacrificed. It should be the aim to show that the two fields are interrelated. The principles of each subject are necessary in the teaching of the other.

There are literally hundreds of ways of accomplishing this goal. It grows on one almost as if it were a game. It gradually becomes part of the student's mental equipment. It comes from many experiences in working together. It does not come from some printed material, but rather from working in the two fields over a period of time.

The program of correlation should begin the first week of school in the seventh grade science class. This is an opportunity to discuss

the proper way to solve problems by the scientific method of thinking:

1. State the problem.
2. Consider various ways of solving the problem.
3. Choose the most logical solution.
4. Test it.
5. Test it over and over again.

Then the student should be reminded that this is the same procedure used in solving a mathematics problem. It should also be pointed out that there is constant comparing, use of dimensions, drawing or making things to scale.

Just recently in one of our eighth grade science classes an aquarium was being constructed. The committee gathered their information on how one should be made and proceeded to work. First they took the measurement of the aquarium and figured the number of gallons of water it would hold. With the knowledge that the aquarium held six gallons of water, they knew they might purchase six one-inch fish for it. Next, they washed the sand, aerated the water to rid it of chlorine, and then permitted the water to stand for 24 hours. Then they were ready to purchase the plants and fish. They were told they might go to the pet shop, do their purchasing and charge their purchases to the school. It was when they returned that I wondered about my success as a math teacher, for Dick, one of the members of the group, came in which much exuberance, saying that not only had they bought fish and plants but that they had also bought a castle that cost scarcely nothing at all. When I asked Dick what he meant by scarcely nothing at all, he said, "You know the school gets a 20% discount and by adding just a few cents more to the 20% discount we got the castle." An example of correlating science and economics to get what you want.

An analysis of the subject reveals that there is hardly a unit of science in which the correlation of math and science is not demonstrated. A few of the examples of correlation are:

WEATHER

1. Reading a thermometer
2. Finding the temperature at which water boils and freezes
3. Keeping a temperature record and finding the average temperature for a weeks time
4. Studying a barometer to learn how air pressure is measured
5. Learning how the barometer is used in predicting weather
6. Studying wind velocity
7. Finding how precipitation is measured
8. Keeping a rainfall chart
9. Transferring C. readings to F. readings and vice versa
10. Studying the hygrometer.

STUDYING THE LIFE CYCLE OF BIRDS, INSECTS, AND FISH

1. Discussing the migration of familiar birds and the distance traveled, height at which they fly
2. Comparing incubation periods of frog, bird and fish eggs

LEARNING HOW TIME IS MEASURED

1. Studying the movement of the earth and the speed at which it travels
2. Drawing diagrams to show the position of the earth as it revolves around the sun
3. Studying the various ways of measuring time and our time belts

STUDYING THE EARTH AND THE FORCES WHICH ARE CHANGING IT

1. Discovering how much of the earth's surface is covered by water
2. Discussing and comparing heights above sea level
3. Discussing ocean depths
4. Discussing size and movements of glaciers

STUDYING THE UNIVERSE

1. Make a scale drawing of the orbits of the planets
2. Card table with light bulb for showing cause of day and night and also seasons
3. Discussion of light years
4. Comparing the size of the various planets

INCIDENTALS

1. Construction of cage for pets
2. Changing linear measure to metric measure
3. Reading electric meters
4. What will 50 cents worth of electricity give you?
5. How much does it cost to heat a home?
6. What fuel is likely to be most economical?
7. Discussion of water and air pressure.

THE NAVY IMPROVES FLIGHT EQUIPMENT

ALLEN LONG
Science Service Aviation Writer

A thin parachute pack hugging the backs of Navy pilots will be knocking about $3,000 off the cost of military airplanes in the future. Comdr. R. A. Bosee, parachute test director at the El Centro, Calif., Naval Air Station, said the parachute requires about half the space of its predecessor. Its reduced bulk, about one and a half inches thick in comparison to the ordinary three-inch pack, saves enough space to account for $3,000 in metal, engineering and construction costs.

A "new look" flight suit for Navy pilots is a snug-fitting nylon garment with a revolutionary harness designed to reduce wear and tear on the pilot in high-speed bailouts. The suit distributes the opening shock of the parachute over the pilot's feet, calves, thighs, trunk and shoulders. Conventional harnesses are strapped around the legs and under the arms. The entire impact of the opening canopy is absorbed in these small areas.

Another suit, now in the testing stage, has a built-in parachute harness to which the pilot plugs his parachute, cockpit shoulder straps and seat belt. This slashes by 75% the time required for him to perform these operations. The built-in harness eliminates the complex cross-straps that get in his way.

PROBLEM DEPARTMENT

CONDUCTED BY MARGARET F. WILLERDING
Harris Teachers College, St. Louis, Mo.

This department aims to provide problems of varying degrees of difficulty which will interest anyone engaged in the study of mathematics.

All readers are invited to propose problems and to solve problems here proposed. Drawings to illustrate the problems should be well done in India ink. Problems and solutions will be credited to their authors. Each solution, or proposed problem, sent to the Editor should have the author's name introducing the problem or solution *as on the following pages.*

The editor of the department desires to serve its readers by making it interesting and helpful to them. Address suggestions and problems to Margaret F. Willerding, Harris Teachers College, St. Louis, Missouri.

SOLUTIONS AND PROBLEMS

Note. Persons sending in solutions and submitting problems for solution should observe the following instructions.

1. Drawings in India ink should be on a separate page from the solution.

2. Give the solution to the problem which you propose if you have one and also the source and any known references to it.

In general when several solutions are correct, the ones submitted in the best form will be used.

Late Solutions

2395. *Bro. Felix John, Philadelphia, Pa.; Charles Plymale, Marietta, Ohio; Clinton E. Jones, Nashville, Tenn.*

2401. *Proposed by A. R. Haynes Tacoma, Wash.*

Prove:

$$\tan^2\frac{\pi}{17}+\tan^2\frac{2\pi}{17}+\cdots+\tan^2\frac{7\pi}{17}+\tan^2\frac{8\pi}{17}=136.$$

Solution by Leon Bankoff, Los Angeles, Calif.

The equation $\tan 17\theta=0$ is satisfied by $\theta=n\pi/17$ where n is any integer or zero. Writing $\tan\theta\equiv t$, and using the equation

$$\tan n\theta=\frac{\binom{n}{1}t-\binom{n}{3}t^3+\cdots}{1-\binom{n}{2}t^2+\binom{n}{4}t^4-\cdots}=0$$

we obtain

$$\tan 17\theta=17t-680t^3+6188t^5-19{,}448t^7+24{,}310t^9-12{,}376t^{11}+2380t^{13}-136t^{15}+t^{17}.$$

Since the factor t corresponds to $\theta=0$, it follows that $\pm\tan\pi/17$, $\pm\tan 2\pi/17$, $\cdots$ $\pm\tan 8\pi/17$ are roots of

$$t^{16}-136t^{14}+2380t^{12}-12{,}376t^{10}+24{,}310t^8-19{,}448t^6+6188t^4-680t^2+17=0$$

Then $\tan^2\pi/17$, $\tan^2 2\pi/17$, $\cdots$ $\tan^2 8\pi/17$ are roots of

$$t^8-136t^7+2380t^6-12{,}376t^5+24{,}310t^4-19{,}448t^3+6188t^2-680t+17=0.$$

Since the sum of the roots is equal to the coefficient of the second term with the sign changed, we have

$$\sum_{n=1}^{8} \tan^2 \frac{n\pi}{17} = 136.$$

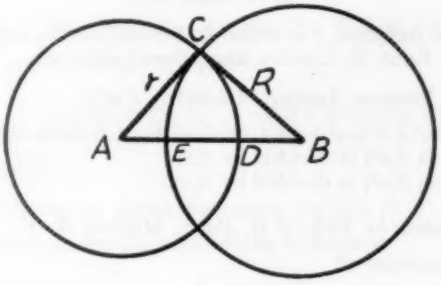
Solutions were also offered by: R. L. Moenter, Fremont, Neb.; Norman Anning, Alhambra, Calif.; C. W. Trigg, Los Angeles City College; Julian H. Braun, China Lake, Calif.; and the proposer.

2402. *Proposed by Leon Bankoff, Los Angeles, Calif.*

The base of a triangle formed by joining the centers of two orthogonal circles and one of their points of intersection, is divided into three segments by the circumferences of the circles. Show that the ellipse with semi-axes equal to the two outer segments is equivalent to twice the area of the incircle of the triangle.

Solution by V. C. Bailey, Evansville, Ind.

Let C be one of the points of intersection of the two orthogonal circles and let their centers be A and B, respectively. Circles A and B cut their line of centers in E and D, respectively.



The inradius of a right triangle is

$$r = \frac{a+b-c}{2}.$$

Then

$$r = \frac{a+b-(a+b-DE)}{2} = \frac{DE}{2}.$$

That is,

$$DE = 2r.$$

Therefore,

$$b = AD+2r \quad \text{and} \quad a = EB+2r.$$

$$a^2+b^2 = c^2.$$

Then

$$(EB+2r)^2+(AD+2r)^2 = (AD+EB+2r)^2.$$

On expanding and simplifying we find that

$$2r^2 = AD \cdot EB.$$

If both sides of the last equation are multiplied by pi, we get

$$2(\pi r^2) = \pi AD \cdot EB.$$

Hence, the theorem is proved.

Solutions were also offered by: R. L. Moenter, Fremont, Neb.; Joseph Kennedy, Greenfield, Ind.; C. W. Trigg, Los Angeles City College; Robert G. Cawley, Dunmore, Pa.; Norman Anning, Alhambra, Calif.; Richard H. Bates, Milford, N. Y.; Herta Freitag, Roanoke, Va.; A. R. Haynes, Tacoma, Wash.; and the proposer.

2403. *Proposed by Gene S. Jamison, St. Charles, Mo.*

A belt 40 inches long (no sag) is around two circular wheels 12 inches center to center. If the radii are in the ratio 9:1 find the radius of the smaller circle.

Solution by V. C. Bailey, Evansville, Ind.

If we let r and $9r$ be the radii of the two wheels, then

$$40=9r\left[2\pi-2\arccos\frac{2r}{3}\right]+2\sqrt{144-64r^2}+r\left[2\arccos\frac{2r}{3}\right].$$

This may be written

$$r=\frac{20-4\sqrt{9-4r^2}}{9\pi-8\arccos 2r/3}.$$

By the method of iteration, r is found to be 0.47 inches, approximately.

C. W. Trigg and Robt. G. Cowley also offered solutions.

2404. *Proposed by Norman Anning, Alhambra, Calif.*

Given that $f(x)=x^4+x^3+x^2+x+1$, find without tedious division:

a) remainder when $f(x^4)$ is divided by $f(x)$,

b) remainder when $f(x^5)$ is divided by $f(x)$.

Solution by Richard H. Bates, Milford, N. Y.

a) $f(x)$ can be expressed as

$$\frac{x^5-1}{x-1}$$

and $f(x^4)$ as

$$\frac{x^{20}-1}{x^4-1}=\frac{(x^{10}+1)(x^5+1)(x^5-1)}{(x^2+1)(x+1)(x-1)}.$$

Upon dividing $f(x^4)$ by $f(x)$:

$$\frac{(x^{10}+1)(x^5+1)(x^5-1)}{(x^2+1)(x+1)(x-1)}\cdot\frac{(x-1)}{(x^5-1)}=\frac{(x^{10}+1)(x^5+1)}{(x^2+1)(x+1)}.$$

An examination of this last fraction reveals that $(x^{10}+1)$ is exactly divisible by (x^2+1) and (x^5+1) is exactly divisible by $(x+1)$; hence the remainder is 0.

b) This part of the solution is by the proposer.

$$f(x^5)=(x^{20}-1)+(x^{15}-1)+(x^{10}-1)+(x^5-1)+1+4$$
$$=\{\text{a multiple of } f(x)\}+5.$$

Solutions were also offered by C. W. Trigg, Los Angeles City College; H. M. Feldman, St. Louis, Mo.; Brother Felix John, Philadelphia, Pa.; A. R. Haynes, Tacoma, Wash.; and the proposer.

2405. *Proposed by Richard Bates, Milford, N. Y.*

Show that

$$\sum_{k=1}^{n}\cos(k\theta-\phi)=0 \qquad \phi<\theta,\ \theta=2\pi/n.$$

Solution by C. W. Trigg, Los Angeles City College

$$\sum_{k=1}^{n} \cos(k\theta-\phi) = \sum_{k=1}^{n} [\cos k\theta \cos\phi + \sin k\theta \sin\phi]$$

$$= \cos\phi \sum_{k=1}^{n} \cos k\theta + \sin\phi \sum_{k=1}^{n} \sin k\theta = 0, \quad \text{since}$$

$$\sum_{k=1}^{n} \cos k\theta = \frac{\cos(n+1)\theta/2 \cdot \sin n\theta/2}{\sin\theta/2} = \frac{\cos(n+1)\pi/n \cdot \sin\pi}{\sin\pi/n} = 0, \qquad n \neq 1,$$

and

$$\sum_{k=1}^{n} \sin k\theta = \frac{\sin(n+1)\pi/n \cdot \sin\pi}{\sin\pi/n} = 0.$$

It will be observed that we have not used the condition $\phi < \theta$, but have imposed the obviously necessary condition, $n \neq 1$.

Solutions were also offered by: Aaron Buchman, Buffalo, N. Y.; R. L. Moenter, Fremont, Neb.; Norman Anning, Alhambra, Calif.; Robert G. Cawley, Dunmore, Pa.; Clinton E. Jones, Nashville, Tenn.; Willis B. Porter, New Iberia, La.; and the proposer.

2406. *Proposed by C. W. Trigg, Los Angeles City College.*

The area of an acute triangle is equal to

$$\frac{r}{2}[(a+b)\cos C + (a+c)\cos B + (b+c)\cos A]$$

where r is the inradius.

Solution by Aaron Buchman, Buffalo, N. Y.

A well known formula for the area of *any* triangle is

$$K = \frac{r}{2}(a+b+c). \tag{1}$$

But it is easily shown that in *any* triangle

$$a = b\cos C + c\cos B \tag{2}$$

$$b = c\cos A + a\cos C \tag{3}$$

$$c = a\cos B + b\cos A \tag{4}$$

Replacing a, b and c in formula (1) by the respective values in (2), (3) and (4), rearranging terms, and collecting coefficients yields the required formula, which is true for *any* triangle.

Note: This formula need *not* be restricted to *acute* triangles.

Solutions were also offered by: Norman Anning, Alhambra, Calif.; Richard H. Bates, Milford, N. Y.; V. C. Bailey, Evansville, Ind.; R. L. Moenter, Fremont, Neb.; Bro. James F. Gray, San Antonio, Texas; Charles H. Butler, Kalamazoo, Mich.; David Rappaport, Chicago, Ill.; A. R. Haynes, Tacoma, Wash.; Brother Felix John, Philadelphia, Pa.; M. D. Bridgess, West Roxbury, Mass.; and the proposer.

STUDENT HONOR ROLL

The Editor will be very happy to make special mention of classes, clubs, or individual students who offer solutions to problems submitted in this department. Teachers are urged to report to the Editor such solutions.

Editor's Note: For a time each student contributor will receive a copy of the magazine in which his name appears.

For this issue the Honor Roll appears below.

2402, 6. *Geoffrey Kandall, Far Rockaway, N. Y.*

2409. *Ann Eaton, Keokuk, Iowa.*

PROBLEMS FOR SOLUTION

2425. *Proposed by Richard H. Bates, Milford, N. J.*

Show that the sum of the first n even integers multiplied by the first n odd integers term by term in reverse is equal to twice the sum of the squares of the first n integers.

2426. *Proposed by Felix John, Philadelphia, Pa.*

In circle O, P is the midpoint of chord AB. Chords RS and TV pass through point P. RV cuts AP at point M, and ST cuts PB at point N. Prove by *high school* geometry that MP equals PN.

2427. *Proposed by Julian H. Braun, Washington, D. C.*

Show that the principle value of arc sin $1/(1+e)$ is given by

$$\frac{\pi}{2}-\sqrt{2e}\left(1-\frac{5e}{12}+\frac{43e^2}{160}-\cdots\right) \qquad \text{for } |e|<1.$$

2428. *Proposed by Julian H. Braun.*

Show that

$$\sec 42^\circ=-\sqrt{3}+\sqrt{5+2\sqrt{5}}.$$

2429. *Proposed by C. W. Trigg, Los Angeles City College, Los Angeles, Calif.*

Reconstruct the division

$$\begin{array}{l} x\ x\,)\,x\ x\ x\ x\ 9\ x\,)\,x\ x\ x\ x\ x \\ \quad x\ x \\ \quad\overline{x\ x\ x} \\ \quad\ \ x\ x \\ \quad\ \overline{\qquad x\ x\ 9} \\ \quad\qquad x\ x\ x \\ \quad\qquad\overline{\qquad\quad x\ x} \\ \quad\qquad\quad\ x\ x \end{array}$$

2430. *Proposed by C. W. Trigg.*

The eye of an observer standing in a ditch is level with the surrounding horizontal terrain. At a distance of x yards he observes a building y yards high, topped by a billboard z yards high, above which w yards of a vertical flag pole project. If y, z, w subtend equal angles at the eye of the observer, then (1) find x, (2) determine the limiting conditions on y and z and (3) ascertain the smallest integer values of x, y, z, w.

HEAT-REPELLENT PAINT

Heat-repellent paint, applied to a building's roof, reportedly may lower interior temperatures as much as 45 degrees. This "air-conditioning" paint reflects heat, light and infrared waves from its white surface. It has been applied successfully to steel, wood, aluminum, glass, asphalt shingle and masonry.

BOOKS AND PAMPHLETS RECEIVED

INTRODUCTION TO MATHEMATICAL STATISTICS, Second Edition, by Paul G. Hoel, *Professor of Mathematics, University of California, Los Angeles.* Cloth. Pages xi+331. 14.5×23 cm. 1954. John Wiley and Sons, Inc., 440 Fourth Avenue, New York 16, N. Y. Price $5.00.

MATHEMATICS IN AGRICULTURE, Second Edition, by R. V. McGee, *Department of Mathematics, The Agricultural and Mechanical College of Texas.* Cloth. Pages x+208. 14×21.5 cm. 1954. Prentice-Hall, Inc., 70 Fifth Avenue, New York 11, N. Y. Price $4.00.

APPLIED ATOMIC ENERGY, by K. Fearnside, M. A., A.M.I.E.E.; E. W. Jones B.Sc., A.Inst.P.; and E. N. Shaw, B.Sc. Cloth. Pages viii+156. 12×18.5 cm. 1954. Philosophical Library, 15 East 40th Street, New York 16, N. Y. Price $4.75.

THEORY OF FUNCTIONS OF A COMPLEX VARIABLE, Volume One, by C. Caratheodory, *Professor, University of Munich,* and Translated by F. Steinhardt, *Instructor, Columbia University.* Cloth. Pages xii+301. 14×23 cm. 1954. Chelsea Publishing Company, 552 West 181 Street, New York 33, N. Y.

INTRODUCTION TO COLLEGE PHYSICS, by Rogers D. Rusk, *Professor of Physics, Mount Holyoke College.* Cloth. Pages xxii+816. 15×23 cm. 1954. Appleton-Century-Crofts, Inc., 35 West 32nd Street, New York 1, N. Y. Price $6.50.

AN ANALYTICAL CALCULUS FOR SCHOOL AND UNIVERSITY, Volume II, by E. A. Maxwell, *Fellow of Queen's College, Cambridge, London.* Cloth. Pages vi+272. 1954. Cambridge University Press, American Branch, 32 East 57th Street, New York 22, N. Y. Price $3.50.

COLLEGE ALGEBRA, by H. G. Apostle, *Grinnell College.* Cloth. Pages xii+432. 13.5×21 cm. 1954. Henry Holt and Company, 383 Madison Avenue, New York 17, N. Y. Price $4.50.

INTERMEDIATE ALGEBRA, by L. J. Adams, *Santa Monica City College, California.* Cloth. Pages xiv+370. 13.5×21 cm. 1954. Henry Holt and Company, 383 Madison Avenue, New York 17, N. Y. Price $3.40.

ESSAYS IN SCIENCE, by Albert Einstein. Cloth. Pages xi+114. 10×16.5 cm. 1934. Philosophical Library, 15 East 40th Street, New York 16, N. Y. Price $2.75.

ELEMENTS OF MATHEMATICS FOR RADIO, TELEVISION AND ELECTRONICS, by Bernhard Fischer and Herbert Jacobs, *Hollywood, California.* Cloth. Pages xxi+569. 13.5×21 cm. 1954. The Macmillan Company, 60 Fifth Avenue, New York 11, N. Y. Price $5.40.

MICROBES AND YOU, by Stanley E. Wedberg, Ph.D., *Associate Professor of Bacteriology, University of Connecticut, Storrs, Connecticut.* Cloth. Pages ix+439. 13.5×21 cm. 1954. The Macmillan Company, 60 Fifth Avenue, New York 11, N. Y. Price $4.50.

THE PERMANENT REVOLUTION IN SCIENCE, by Richard L. Schanck, *Chairman, Department of Sociology, Bethany College, Lecturer, Carnegie Institute of Technology.* Cloth. Pages xvi+112. 13.5×21.5 cm. 1954. Philosophical Library, Inc., 15 East 40th Street, New York 16, N. Y. Price $3.00.

A PHILOSOPHICAL STUDY OF THE HUMAN MIND, by Joseph Barrell, *Professor of Philosophy, Beloit College, Beloit, Wisconsin.* Cloth. Pages xii+575. 13.5×21.5 cm. 1954. Philosophical Library, Inc., 15 East 40th Street, New York 16, N. Y. Price $6.00.

HEALTH AND SAFETY FOR YOU, by Harold S. Diehl, *Dean of the Medical Sciences and Professor of Preventive Medicine and Public Health, Minneapolis, Minnesota,* and Anita Duncan Laton, *Professor of Health and Hygiene, San Jose State College, San Jose, California.* Cloth. Pages xii+515. 16×23.5 cm. 1954. McGraw-Hill Book Company, Inc., 330 West 42nd Street, New York 36, N. Y. Price $3.75.

MINNESOTA'S ROCKS AND WATERS, A GEOLOGICAL STORY, by George M. Schwartz, *Director of the Minnesota Geological Survey,* and George A. Thiel, *Chairman of the University Geology Department, Minneapolis, Minnesota.* Cloth. Pages xviii+366. 15×23.5 cm. 1954. University of Minnesota Press, Minneapolis 14, Minn. Price $4.00.

PHYSICS: A BASIC SCIENCE, Third Edition, by Elmer E. Burns, *Instructor of Physics (Emeritus), Austin High School, Chicago, Illinois;* Frank L. Verwiebe, *Research Physicists, Applied Physics Laboratory, The Johns Hopkins University, Baltimore, Maryland;* Herbert C. Hazel, *Formerly Head of Science Department, Bloomington High School, Bloomington, Indiana;* and Gordon E. Van Hooft, *Assistant in Secondary Curriculum, New York State Education Department, Albany, New York.* Cloth. Pages xi+546. 16×24 cm. 1954. D. Van Nostrand Company, Inc., 250 Fourth Avenue, New York 3, N. Y. Price $4.12.

EMERGING PRACTICES IN MATHEMATICS EDUCATION. TWENTY-SECOND YEARBOOK OF THE NATIONAL COUNCIL OF TEACHERS OF MATHEMATICS. Cloth. Pages xiv+434. 15×23 cm. 1954. National Council of Teachers of Mathematics, 1201 Sixteenth Street, N.W. Washington 6, D. C. Price $4.50. To Members of the Council $3.50.

EDUCATIONAL PSYCHOLOGY, by Lee J. Cronbach, *University of Illinois, Urbana, Illinois.* Cloth. Pages xxvii+628. 15.5×23 cm. 1954. Harcourt, Brace and Company, 383 Madison Avenue, New York 17, N. Y.

ANALYTIC GEOMETRY, by Gordon Fuller, *Professor of Mathematics, Texas Technological College.* Cloth. Pages ix+205. 15×23 cm. 1954. Addison–Wesley Publishing Company, Inc., Cambridge 42, Mass. Price $3.85.

CALCULUS, by George B. Thomas, Jr., *Associate Professor of Mathematics, Massachusetts Institute of Technology.* Cloth. 614 pages. 15×23 cm. 1953. Addison-Wesley Publishing Company, Inc., Cambridge 42, Mass. Price $6.50.

A FIRST YEAR OF COLLEGE MATHEMATICS, Second Edition, by Raymond W. Brink, Ph.D., *Professor of Mathematics, University of Minnesota.* Cloth. Pages xvi+725. 14×21.5 cm. 1954. Appleton-Century-Crofts, Inc., 35 West 32nd Street, New York 1, N. Y. Price $5.00.

THE SOPHISTS, by Mario Untersteiner. Translated from the Italian by Kathleen Freeman. Cloth. Pages xvi+368. 13.5×21.5 cm. 1954.

THE ZOO COMES TO YOU, by Captain Burr W. Leyson, *Assistant Curator, Department of Education, New York Zoological Society, New York,* and Ruth Manecke, *Educational Assistant at the Bronx Zoo, New York,* Cloth. 88 pages. 18×24 cm. 1954. E. P. Dutton and Company, Inc., 300 Fourth Avenue, New York 10, N. Y. Price $2.95.

SCHOOL FACILITIES FOR SCIENCE INSTRUCTION. John S. Richardson, Editor. Cloth. Pages viii+266. 21×28 cm. 1954. The National Science Teachers Association, 1201-16th Street, N.W., Washington 6, D. C.

ALGEBRA FOR COLLEGE STUDENTS, Revised Edition, by Jack R. Britton, *Professor of Applied Mathematics,* and L. Clifton Snively, *Assistant Professor of Applied Mathematics,* both of the *University of Colorado, Boulder, Colorado,* Cloth.

Pages xiii+537. 14×21.5 cm. 1954. Rinehart and Company, Inc., 232 Madison Avenue, New York 16, N. Y. Price $4.25.

THROUGH THE MAGNIFYING GLASS, by Julius Schwartz, *Bureau of Curriculum Research of the Board of Education, New York City.* Cloth. 142 pages. 13.5×20.5 cm. 1954. Whittlesey House, McGraw-Hill Book Company, Inc., 330 West 42nd Street, New York 36, N. Y. Price $2.50.

MATHEMATICS FOR THE SECONDARY SCHOOL, by William David Reeve (Retired) *Teachers College, Columbia University, New York.* Cloth. Pages xii+547. 15×23.5 cm. 1954. Henry Holt and Company, 383 Madison Avenue, New York 17, N. Y. Price $5.95.

LET'S DRIVE RIGHT, by Maxwell Halsey, *Executive Secretary of the Michigan Safety Commission, Lansing, Michigan.* Cloth. 496 pages. 18.5×24 cm. 1954. Scott, Foresman and Company, 433 East Erie Street, Chicago 11, Ill. Price $3.48.

THE HANDBOOK OF PRIVATE SCHOOLS. AN ANNUAL DESCRIPTIVE SURVEY OF INDEPENDENT EDUCATION. Thirty-Fifth Edition. Cloth. 1232 pages. 11.5 ×18 cm. 1954. Porter Sargent, 11 Beacon Street, Boston 8, Mass. Price $8.00.

SCIENCE FOR EVERYDAY USE, Third Eidtion, by Victor C. Smith, *Department of General Science, Ramsey Junior High School, Minneapolis, Minnesota,* and B. B. Vance, *Science Department, Kiser High School and Associate Professor of Sciences and Education, The University of Dayton, Dayton, Ohio.* Cloth. Pages xiii+737. 14.5×22.5 cm. 1954. J. B. Lippincott Company, 333 West Lake Street, Chicago 6, Ill.

BIOLOGY FOR YOU, Third Edition, by B. B. Vance, *Chairman of Sciences and a Teacher of Biology at Kiser High School, Dayton, Ohio,* and D. F. Miller, *Chairman of the Zoology and Entomology Department at the Ohio State Univerity, Columbus, Ohio.* Cloth. 652 pages. 16×22.5 cm. 1954. J. B. Lippincott Company, 333 West Lake Street, Chicago 6, Ill.

CHEMISTRY FOR THE NEW AGE, Revised Edition, by Robert H. Carleton, *Executive Secretary of the National Science Teachers Association;* Floyd F. Carpenter, *Principal of Stivers High School,* Dayton, Ohio; and R. W. Woline, *Instructor in Chemistry in the Oak Park and River Forest High School, Oak Park, Illinois.* Cloth. Pages xiv+688. 14.5×22.5 cm. 1954. J. B. Lippincott Company, 333 West Lake Street, Chicago 6, Ill.

PHYSICS FOR THE NEW AGE, Revised Edition, by Robert H. Carleton, *Executive Secretary of the National Science Teachers Aossciation;* Harry H. Williams, *Assistant Principal and Head of the Science Department of the Horace Mann School for Boys, New York City;* and Mahlon H. Buell, *Instructor in Physics, Physical Science, and Photography, Senior High School, Ann Arbor, Michigan.* Cloth. Pages xvi+656. 14.5×22.5 cm. 1954. J. B. Lippincott Company, 333 West Lake Street, Chicago 6, Ill.

SOLID GEOMETRY, by Virgil S. Mallory. *Professor of Mathematics and Instructor in the College High School, State Teachers College, Montclair, New Jersey, and* Chauncey W. Oakley, *Head of the Department of Mathematics, High School, Manasquan, New Jersey.* Cloth. Pages ix+252. 13×20.5 cm. 1954. Benj. H. Sanborn and Company, 5559 Northwest Highway, Chicago 30, Ill.

TRIGONOMETRY, by William L. Hart, *Professor of Mathematics, University of Minnesota, Minneapolis, Minnesota.* Cloth. Pages vi+230+130. 14.5×22.5 cm. 1954. D. C. Heath and Company, 285 Columbus Avenue, Boston 16, Mass. Price $3.75.

TEACHING ARITHMETIC IN GRADES I AND II, by George E. Hollister, *Professor of Elementary Education, University of Wyoming, Laramie, Wyoming,* and Agnes

G. Gunderson, *Professor of Elementary Education, University of Wyoming, Laramie, Wyoming.* Cloth. 168 pages. 15.5×21.5 cm. 1954. D. C. Heath and Company, 285 Columbus Avenue, Boston 16, Mass. Price $2.50.

FUNDAMENTALS OF COLLEGE MATHEMATICS, by John C. Brixey and Richard V. Andree, *The University of Oklahoma, Norman, Oklahoma.* Cloth. Pages xiv+609. 16×23.5 cm. 1954. Henry Holt and Company, 383 Madison Avenue, New York 17, N. Y. Price $5.90.

MASTER'S THESES IN SCIENCE, 1952. Edited by Barton Bledsoe, *Bibliographic Branch, Department of the Army Library.* Cloth. 252 pages. 14.5×22.5 cm. 1954. Biblio Press, Washington 5, D. C.

SCIENCE FOR HERE AND NOW, by Herman and Nina Schneider. Grade 2. Cloth. 222 pages. 14.5×22 cm. 1954. D. C. Heath and Company, 285 Columbus Avenue, Boston 16, Mass. Price $1.92.

ALGEBRA: ITS BIG IDEAS AND BASIC SKILLS, Book I, by Daymond J. Aiken, *Head of Mathematics Department, Lockport Township High School, Lockport, Illinois;* and Kenneth B. Henderson, *Professor of Mathematics Education, University of Illinois.* Cloth. Pages xi+419. 15×23 cm. 1954. McGraw-Hill Book Company, Inc., 330 West 42nd Street, New York 36, N. Y. Price $2.84.

STONE AGE ECONOMISTS IN THE ATOMIC AGE, by Olaf Nelson. Cloth. Pages ix+160. 15×23 cm. 1954. Pagent Press, Inc., 130 West 42nd Street, New York, N. Y. Price $3.00.

SCIENCE FAR AND NEAR, by Herman and Nina Schneider. Cloth. 282 pages. 14.5×22 cm. 1954. D. C. Heath and Company, 285 Columbus Avenue, Boston 16, Mass. Price $2.20.

BETTER TEACHING THROUGH ELEMENTARY SCIENCE, by Julian Greenlee, *Professor of Education, Florida State University.* Paper. Pages xii+204. 13.5 ×21.5 cm. 1954. Wm. C. Brown Company, Dubuque, Iowa. Price $2.50.

FREE AND INEXPENSIVE LEARNING MATERIALS. Sixth Edition. Paper. Pages viii+216. 13.5×21.5 cm. 1954. Division of Surveys and Field Services, George Peabody College for Teachers, Nashville 4, Tenn. Price $1.00.

TEACHING MATERIALS FOR MATHEMATICS CLASSES, Compiled by Kenneth P. Kidd, *University of Florida, Gainesville, Florida,* and Kenneth E. Brown, *Specialist for Mathematics, Office of Education, Washington, D. C.* Paper. Pages v+36. 20×27 cm. Circular No. 399 may be secured free by writing directly to the Publications Section, Office of Education, Department of Health, Education, and Welfare, Washington, 25, D. C.

CIVIL DEFENSE AND ATOMIC WARFARE. A SELECTED READING LIST. Prepared by the U. S. Atomic Energy Commission for the Federal Civil Defense Administration. Paper. Pages iv+48. 15×28 cm. Superintendent of Documents, U. S. Government Printing Office, Washington 25, D. C. Price 25 cents.

MANPOWER RESOURCES IN MATHEMATICS, A Study Conducted Jointly by The National Science Foundation and The United States Department of Labor, Bureau of Labor Statistics. Paper. Pages iv+22. 20×26 cm. Superintendent of Documents, U. S. Printing Office, Washington 25, D. C. Price 20 cents.

PHYSICS ACTIVITIES, by Robert H. Carleton, *Executive Secretary, National Science Teachers Association, Washington, D. C.;* Harry H. Williams, *Assistant Principal and Head of the Science Department, Horace Mann School for Boys, New York City;* Mahlon H. Buell, *Head of Physics Department, Ann Arbor High School, Ann Arbor, Michigan;* and Frederick W. Schuler, *Physics Department, West High School, Madison, Wisconsin.* Paper. 224 pages. 18.5×26 cm. 1954. J. B. Lippincott Company, 333 West Lake Street, Chicago 6, Ill.

BIOLOGY ACTIVITIES, by B. B. Vance, *Science Department, Kiser High School and Asistant Professor of Biology and Education, The University of Dayton, Dayton, Ohio;* C. A. Barker, *Former Instructor of Biology, Steele High School, Dayton, Ohio;* and D. F. Miller, *Zoology and Entomology Department, The Ohio State University, Columbus, Ohio.* Paper. 254 pages. 18.5×26 cm. 1954. J. B. Lippincott Company, 333 West Lake Street, Chicago 6, Ill.

CHEMISTRY ACTIVITIES, by Robert H. Carleton, *Executive Secretary of the National Science Teachers Association, Washington, D. C.;* John H. Woodburn, *Assistant Professor of Science, Illinois State Normal University, Normal, Illinois;* and Thaddeus H. Elder, Jr., *Formerly Chemistry Teacher, Hyattsville High School, Hyattsville, Maryland.* Paper. 256 pages. 18.5×26 cm. 1954. J. B. Lippincott Company, 333 West Lake Street, Chicago 6, Ill.

SCIENCE TEACHING: AT THE CROSSROADS, by Richard Van Fleet, *Personnel Manager, Central Laboratories, General Foods.* This Talk Was Delivered before a Meeting of the New Jersey Science Teachers Association at Rutgers University, May 1, 1954. Paper. 15 pages. 10×21 cm.

TECHNICIAN'S GUIDE TO TV PICTURE TUBES, by Ira Remer. Paper. Pages v +154. 14×21.5 cm. 1954. John F. Rider Publisher, Inc., 480 Canal Street, New York 13, N. Y. Price $2.40.

HOW TO LOCATE AND ELIMINATE RADIO AND TV INTERFERENCE, by Fred D. Rowe, *Northern California Electrical Bureau.* Paper. Pages v+122. 14×21.5 cm. 1954. John F. Rider Publisher, Inc., 480 Canal Street, New York 13, N. Y. Price $1.80.

FUNDAMENTALS OF TRANSISTORS, by Leonard M. Krugman, B.S., M.S., P.E., *Signal Corps Engineering Laboratories.* Paper. 144 pages. 13.5×21.5 cm. 1954. John F. Rider Publisher, Inc., 480 Canal Street. New York 13, N. Y. Price $2.70.

A TREATISE ON CONIC SECTIONS, by George Salmon, D.D., D.C.L., LL.D., F.R.S., *Regius Professor of Divinity in the University of Dublin.* Sixth Edition. Paper. Pages xv+399. 11.5×20.5 cm. Chelsea Publishing Company, 552 West 181 Street, New York 33, N. Y. Price $1.94.

INTRODUCTION TO THE THEORY OF ERROR, by Yardley Beers, *Associate Professor of Physics, New York University.* Paper. Pages vi+65. 13.5×19.5 cm. 1953. Addison-Wesley Publishing Company, Inc., Cambridge 42, Mass. Price $1.25.

HIGH SCHOOL RETENTION BY STATES, by Walter H. Gaumnitz, *Specialist for Small and Rural High Schools.* Circular No. 398. June 1954. Paper. Pages iv+19. 19.5×27 cm. U. S. Department of Health, Education, and Welfare, Office of Education, Washington 25, D. C.

FIFTY YEARS OF GRADUATE EDUCATION AT THE UNIVERSITY OF WISCONSIN, Prepared by the Deans of the Graduate School, Associate Dean W. J. Brogden, Associate Dean J. Homer Herriott, and Dean Conrad A. Elvehjem. Paper. 72 pages. 21.5×28 cm. The University of Wisconsin Graduate School, Madison 6, Wis.

MATHEMATICS EDUCATION RESEARCH STUDIES—1953, Prepared by Kenneth E. Brown. Paper. 22 pages. 20.5×27 cm. Copies may be secured free by writing to the Publications Section, Office of Education, U. S. Department of Health, Education, and Welfare, Washington 25, D. C.

ROTATING ELECTRICAL MACHINERY. Paper. 256 pages. 21.5×28 cm. 1954. Crow Electri-Craft Corp., 1102 Shelby Street, Vincennes, Ind. Price $3.50.

MANUAL FOR CONSTRUCTION AND USE OF SLIDE RULE, by Kenneth P. Kidd,

Ph.D., *College of Education, University of Florida.* Paper. 4 pages. 21.5×28 cm. Price 25 cents each. In lots of 10 or more, 20 cents each.

THE PROFESSION OF ENTOMOLOGY 1854–1954, by Centennial Commemoration Committee, David G. Hall, Chairman. Section One, General Information. 24 pages. Section Two, News and Feature Information. 139 pages. Section Three, Visuals. 43 pages. Paper. 21×28 cm. U. S. Department of Agriculture, Washington 25, D. C.

BOOK REVIEWS

INTRODUCTION TO MATHEMATICAL STATISTICS by Paul G. Hoel, *Professor of Mathematics, University of California, Los Angeles.* Second Edition. Cloth, Pages xi+331. 15.5×23.5 cm. 1954. John Wiley and Sons, Inc., 440-4th Ave., New York 16, N. Y. Price $5.00.

This is the second edition of a text giving a mathematical treatment of elementary statistical theory. While the primary emphasis is placed upon theory, by no means are applications completely disregarded. The author has made a definite effort to make the work readable to a student with a background of only elementary calculus, and with a few minor exceptions, seems to have attained his objective.

In this revision the number of pages has been increased by more than twenty five per cent. A chapter on probability, essentially the first chapter in the text, brings together material which was scattered through the first edition, and in addition expands the treatment very appreciably. This chapter is followed by another new chapter entitled "Nature of Statistical Methods" much of which is new and distinctly of value. Very definitely this is not a mere reprinting, but a somewhat radical revision. As a single illustration, one might consider the material on correlation and regression, which in the opinion of this reviewer is a very definite improvement.

Perhaps the most striking thing about this new edition is not the new material, but the extent to which the order of topics has been altered from the first edition. For this reason, the instructor who has used certain chapters from the earlier edition as a basis for a short course of perhaps one semester will find his material widely scattered in this new edition, and may even (because of the secquene of material) find it difficult to use the second edition as a text if his course content is to remain the same. There are many more exercises; however, in general, exercises in the first edition have not been changed.

CECIL B. READ
University of Wichita

DIFFERENTIAL AND INTEGRAL CALCULUS by Clyde E. Love, Ph.D., *Professor Emeritus of Mathematics in the University of Michigan* and Earl D. Rainville, Ph.D., *Professor of Mathematics in the University of Michigan.* Fifth Edition. Cloth. Pages xiv+526. 16.5×24 cm. 1954. The Macmillan Company, New York, N. Y. Price $5.75.

This is the fifth edition of a well known text. Professor Love requested Professor Rainville to take over the revision of the previous edition. Those who liked the old text will doubtless be pleased with this revision; those not satisfied with the previous work will not find sufficient changes to alter their opinion. The text does not introduce integration as early as is desired by some teachers, for example such applications as volumes of solids of revolution, centroids, fluid pressure, and work do not appear until approximately page 300. In this revision the number of problems has been slightly increased, and in many cases have been changed. As a rough estimate, however, more than one third of the exercises are the same as in the previous edition. No omissions of content were noted except in the case of reduction formulas for integration.

Changes which pleased this reviewer include an improved and expanded treatment of Newton's method, the addition of material on work, an integral test for convergence of infinite series, material on the circle of curvature, and a discussion of oblique and curvilinear asymptotes. There are a couple of sections which discuss the evaluation of iterated integrals by change of coordinates and by inversion of order—material not usually found in a first text in calculus. The treatment of limits in the first chapter seems improved, and for those who wish to make use of the material, there is a short appendix giving a more extensive treatment of limits in a more rigorous manner than usually found.

Another improvement is the use of capital letters to indicate the principal value of an inverse trigonometric function. One might question, however, the authors' use of the equality signs in all cases when defining the principal value. (Is there, for example, a value of x for which Arc csc $x=0$, as is implied by the definition?)

In the illustrative examples relating to curve tracing (pages 201, 212, 216) the statement is made that there is no symmetry. This is true, but it is doubtful if the student has tested for all possible types of symmetry if he follows the text, which only discusses symmetry with respect to the coordinate axes and the origin.

In the opinion of this reviewer, the changes, although not many in number, have resulted in an improvement in an already excellent text.

CECIL B. READ

METHODS OF ALGEBRAIC GEOMETRY by W. V. D. Hodge, Sc.D., F.R.S., *Lowndean Professor of Astronomy and Geometry, and Fellow of Pembroke College, Cambridge* and D. Pedoe, B.Sc. (Lond.), Ph.D. (Cantab.), *Sometime Charles Kingsley Bye-Fellow of Magdalene College, Cambridge*. Volume III, Book V: Birational Geometry. Cloth. Pages x+336. 14.5×22 cm. 1954. Cambridge University Press, 32 East 57th St., New York 22, N. Y. Price $7.50.

The authors state in the preface that this book is written to meet the needs of geometers trained in the classical methods of algebraic geometry who are anxious to acquire the new and powerful tools provided by modern algebra. For such readers, it would seem the objective has been achieved. The book is definitely not one for the beginner; mathematical maturity and background needed are at least of a graduate level. The first eighty pages are devoted to an extension of certain concepts developed in an earlier volume of the set (in fact, one should have the previous volumes for facility in following the argument). Some idea of the content may be indicated by listing a few section headings, selected somewhat at random: Prime ideals and primary ideals; quotient rings; algebraic varieties in affine space; ordered Abelian groups; valuations of algebraic function fields; some Cremona transformations; the reduction of the singularities of an algebraic variety.

Unquestionably this book will be of interest to a somewhat limited number of readers of SCHOOL SCIENCE AND MATHEMATICS; for this group it is somewhat unique in its field; the concern with method primarily means that often the authors deliberately present alternative methods.

CECIL B. READ

NUMBER, THE LANGUAGE OF SCIENCE, A Critical Survey Written for the Cultured Non-Mathematician, by Tobias Dantzig. Ph.D., *Professor of Mathematics, University of Maryland, Retired*. Fourth Edition, Revised and Augmented. Cloth. Pages ix+340. 14.5×21.5 cm. 1954. The Macmillan Company, New York, N. Y. Price $5.00.

Since many readers are familiar with the original or an earlier edition of this well known text, it is important to note the changes. A rather careful comparison with the second edition (1933) shows that except for very minor changes, the first twelve chapters (Part One of the fourth edition) are identical. Twelve illustrations, in all but one case portraits of mathematicians, which appeared in the second edition are not found in the fourth edition. In most cases the changes are

revisions of wording; an erroneous statement on page 157 is now corrected, on page 46 reference is made to perfect numbers discovered by use of modern high speed calculating machines.

The author points out in the preface that Part Two—Problems, Old and New—is for all intents and purposes a new book. The discussion here tends to be somewhat more technical.

It is hard to make a recommendation about this edition. Certainly if your school library does not have a copy of an earlier edition, this is almost a *must*. On the other hand, where the library does contain a copy of a previous edition (such as the second, mentioned above) the matter of funds available may make it doubtful as to whether one is justified in purchasing a new edition containing about eight pages of new material. This might be more to the point in the case of a high school library than a college or university library.

It is hard to think of any book which so well presents the development of the concept of number; it will be of interest not only to the student of mathematics but to the general reader as well.

There is a typographical error on pages 203–204; on page 113 the reference (unchanged from earlier editions) should be to *Second* Chronicles; a footnote on page 168 referring to Appendix 20 is puzzling since there is no appendix. These are of course minor points, this is a book which every teacher of mathematics should be familiar with, and once having seen the book will wish a personal copy.

CECIL B. READ

ALGEBRA FOR PROBLEM SOLVING BOOK 2, by Julius Freilich, *Chairman, Mathematics Department, Brooklyn Technical High School, New York*, Simon L. Berman, *Chairman, Mathematics Department, Stuyvesant High School, New York*, and Elsie Parker Johnson, *Formerly Chairman, Mathematics Department, Oak Park and River Forest High School, Oak Park, Illinois*, Cloth. Pages 511. 16.5 cm by 23 cm. 1953. Houghton Mifflin Co. Cambridge, Massachusetts. $3.20.

One can not open Book 2, *Algebra for Problem Solving* without noting that it is a "red letter" edition. The key words and other key symbols to the idea being presented are in red type. Red ink is also used in many of the pictures, graphs, etc.

The text gives broad coverage of the second course in Algebra. The authors divide the subject into four parts: Refresher Course in Basic Concepts; From Elementary to Intermediate Algebra; New Skills and Broadened Concepts; Intermediate Algebra Extended. This is ample material for a full year course.

As the above remarks suggest, the work moves from rather elementary topics to rather advanced topics. This is also true within each topic. For the more gifted one finds stimulating sections—"Extra for Experts," "Advanced Problems," and "Optional Sections."

A high school junior should be able to read the explanations and developments. They are well done—clear, simple and complete. There is more reading than in the average text. The style is a little more like a book to be read.

In closing the text one reflects that the functional concept has been in focus. One remembers that graphical work has been emphasized.

FOREST MONTGOMERY
Lyons Township High School and Junior College
La Grange, Illinois

MATHEMATICS AND STATISTICS FOR ECONOMISTS, by Gerhard Tintner, *Professor of Economics, Mathematics, and Statistics, Iowa State College*. Cloth. Pages xiv+363. 6×9 inches. 1953. Rinehart & Comapny, Incorporated. Price $6.50.

The author of this text feels that, for serious study of economics, a thorough background in college mathematics is necessary. This book was written to meet

the specific needs of those economics students who lack sufficient mathematical training.

It is definitely a mathematics text but the illustrative and problem material is, almost without exception, taken from the field of economics. The mathematical subject matter covers the wide range from elementary algebra to integral calculus. The treatment is, of course, less rigorous than that generally found in pure mathematics texts; but the explanations and illustrations are good and the problems plentiful. Some of the topics included are: systems of linear equations,, quadratic equations, logarithms, progressions, linear difference equations, elements of differential and integral calculus, probability, frequency distributions, sampling, tests of hypotheses and correlation. Answers to the odd-numbered problems are given at the back of the book.

It is difficult to evaluate a book of this nature. On the whole, it seems to be an excellent text for the intended purpose, but a question arises as to whether or not students who have little mathematical training will be able to digest so large a chunk of mathematics at one sitting. I suppose the answer lies in how the book is to be used.

WILLIAM STRETTON
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COLLEGE ALGEBRA, by Jack R. Britton, *Professor of Applied Mathematics, The University of Colorado;* L. Clifton Snively, *Assistant Professor of Applied Mathematics, The University of Colorado.* Cloth. Pages x+502. 14.5×21 cm. 1953. Rinehart and Company, Inc. 232 Madison Avenue, New York 16, N. Y. Price $5.00.

This book is based in part on the authors' *Algebra for College Students* with the earlier portions of the work in this book somewhat condensed. Also, as additional material beyond that covered in the *Algebra for College Students*, the authors have included the classical solutions of the cubic and the quartic equation, the trigonometric solution of the cubic in the case of three real roots, a chapter on partial fractions, and one on finite differences.

Although the book seems fairly traditional, there are some topics which are especially well done. Chapter XI consists of an excellent but brief discussion of certain aspects of computation with approximate numbers. In chapter XII on logarithms, the work on the change of base and interpolation are treated with unusual clarity. A short explanation of the use of tables of natural logarithms has been included. In chapter XV, the authors have inserted a brief exposition of vectors and have made use of the vector representation in order to give the student a more concrete feeling for complex numbers. The chapter on partial fractions is a little different from the usual run of texts. The idea of splitting off one partial fraction at a time has been made the basis for the theory of partial fractions.

The definition of function is delayed until page 85 while the variable is mentioned on page 86. This seemed, to the reviewer, to be an unnecessary delay in introducing such a basic concept.

The book contains 120 sets of carefully graded exercises. A wide range of well selected applied problems is one of the best features of the book. Answers to most of the odd-numbered problems are given.

The book is worthy of careful consideration when selecting a text in college algebra.

CLYDE T. MCCORMICK
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Normal, Illinois

MATHEMATICS FOR THE CONSUMER, Revised, by Francis G. Lankford Jr., Raleigh Schorling, and John R. Clark. Cloth. Pages ix+438. 16×24 cm. 1953. World Book Company. Yonkers-on-Hudson, New York. Price $2.76.

The material in this book is organized into seven units covering as many aspects of consumer problems: How to Handle Statistics, Better Buymanship, Using Consumer Credit, At Home on the Job, Investments, Greater Security (Insurance), and Taxation. The introduction to each unit outlines actual consumer situations that can aid the teacher in arousing interest and making a smooth transition into the specific problems that follow.

In the Revised Edition the necessary changes have been made to bring prices of commodities into line with present-day conditions and to give up-to-date information on Federal Taxes, United States Savings Bonds, and Social Security regulations. Provision is made in the book for a variety of local investigations that serve to tie classroom exercizes to current local conditions.

The book is well supplied with exercises which are built around everyday use of mathematics. Also, inventory and practice tests with answers and various other tests are included in the text.

The text presents a terminal course in general mathematics that should meet the needs of many schools by giving real meaning to the development of the arithmetic processes on an adult level.

CLYDE T. MCCORMICK

MATHEMATICS IN AGRICULTURE, by R. V. McGee, *Department of Mathematics, The Agricultural and Mechanical College of Texas*. Cloth. Pages x+208. 14.5 ×21.5 cm. 1954. Prentice-Hall, Inc., 70 Fifth Avenue, New York, N. Y. Price $4.00.

This is a practical book on mathematics designed especially for persons interested in agriculture. Improvements undertaken in the revision include the following: increased emphasis on problems arising from the use of machine power and electricity on the farm; modification of problem data in the light of current prices and practices; expansion of the treatment of algebra; and the screening of problems on the basis of their significance in agriculture. In checking many of the problems the reviewer did not find a single item which seemed to be priced out of line with current prices.

The book contains a wealth of problems with appropriate and realistic settings and much reliable material pertaining to various phases of agriculture. Many of the problems contain interesting and useful formulas. For example, on page 19 the following problem is given: A formula developed by the Minnesota Agricultural Experiment Station for the approximate annual depreciation, d, of milk cows is $d=0.015V_0+0.16\frac{2}{3}(V_0-V_n)$, where V_0 is the number of dollars in the original value, and V_n is the number of dollars in the final "block" value. Counting the "block" value of a $175 cow as $70, find the approximate annual depreciation.

A store of useful agricultural data is contained in the 12 sets of tables at the end of the book. The book has a rather complete table of contents and is carefully indexed. The book is clearly and attractively written and has many well c'osen illustrative examples. Few if any answers to problems are provided in the text except for the illustrative examples. It appears to be teachable and should be given serious consideration by any one looking for this type of text.

CLYDE T. MCCORMICK

INTERMEDIATE ALGEBRA, by E. J. Adams, *Santa Monica City College*. Cloth, Pages xiv+370. 14.5×21 cm. 1954. Henry Holt and Company, New York. N. Y. Price $3.40.

This textbook had its origin in 1942 as revision of Rietz and Crathrone's *Introductory College Algebra*.

This book is of the traditional type in intermediate algebra. There is a special emphasis on the applications of algebra to physics, chemistry, engineering, geometry, and the mathematics of finance. A chapter on permutations, combinations, and probability and one on an introduction to the theory of equations appears at the end of the book.

An unusually large quantity of drill exercises and problem material has been included so that the text may be used with equal facility in classes that meet three, four, or five times per week.

The book contains a rather complete table of contents, a list of signs and symbols, the Greek Alphabet, and an index. Answers to odd-numbered problems are given.

It is clearly written and is an excellent textbook for a traditional course in intermediate algebra.

CLYDE T. MCCORMICK

AN INTRODUCTION TO THE HISTORY OF MATHEMATICS, by Howard Eves, *Professor of Mathematics, State University of New York*. Cloth. Pages xv+422, 23.5 ×16 cm. 1953. Rinehart and Company, Inc., New York, N. Y. Price $6.00.

This is an excellent book on the history of mathematics. The organization and content of the text makes it one of the most teachable books in this area of mathematics. But more than this, for the causal reader who is interested in the growth of mathematics, it makes interesting reading. Written for an undergraduate course in the history of mathematics, it is restricted primarily to "elementary" mathematics, that is, mathematics through the beginnings of the calculus. The casual reader will find that a knowledge of simple arithmetic, secondary school algebra, geometry, and trigonometry is sufficient for the first nine chapters. A knowledge of the rudiments of analytic geometry and a few basic concepts of the calculus is sufficient for understanding chapters X and XI.

The book points out in an interesting manner the chronological growth of mathematics from its primitive form to the possibilities of the modern mathematics superstructure. An important innovation of the text is the inclusion of problems. At the end of each chapter is found a set of Problem Studies, each problem containing a number of associated problems and questions. This increases its usefulness for class discussion and helps the student to crystallize many important historical concepts. Rather than just tell a student that the ancient Greeks solved quadratics geometrically, the student is given an opportunity to solve quadratics by the method used by the ancient Greeks.

Oftentimes there is difficulty in pronouncing Hindu and Arabic names. The text helps the reader to circumvent this difficulty in the introductory chapter as well as in the chronological table given at the end of the text.

Part I of the text includes the important contributions of the Babylonians, Egyptians, Greeks, Hindus, and Arabs. Part II includes the later European mathematics, with special emphasis on the seventeenth century. The final chapter includes a description of the trend of mathematical development from the intuition and formalism of the eighteenth century to the rigor of the twentieth century.

As a textbook in the history of mathematics for teacher education purposes it is outstanding.

T. E. RINE
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Normal, Illinois

COLLEGE ALGEBRA, by H. G. Apostle, *Grinnell College*. Cloth. Pages xii+432, 21.5×14 cm. 1954. Henry Holt and Company, New York, New York. Price $4.50.

This college algebra text is written for use in the freshman year of college or technical schools. The emphasis of the text is on the order and devices of presentation which seem most likely to encourage the student's interest in the subject and to offer a general approach to the study of any science. The author makes use of the inductive method to connect the ideas and operations of algebra with their meaningful applications in the physical world. The terminology is relatively simple. The text proceeds gradually from the easier to the more difficult topics. The conditions under which operational ideas are valid are fairly clearly stated.

There is a fair degree of rigor in the deductions of algebraic theorems. In the first chapter the definition of science and the position of algebra as a science is given consideration. This is a relatively new feature in college algebra textbooks. In the second chapter the subject-matter of college algebra is considered in detail. Quantity is distinguished from number and both are distinguished from the symbols which represent them. In the third chapter axioms are laid down and theorems are deduced from them. The purpose of the author here, evidently, is to present to the student some idea of rigor in mathematical reasoning. Chapters four through twelve include the usual topics found in most "intermediate algebra" texts. The remaining chapters include a good treatment of such topics as inequalities, ratio, proportion and variation, permutations and combinations, probability, the binomial theorem, mathematical induction, logarithms, complex numbers, theory of equations, determinants, and partial fractions. In this latter half of the text the chapters are written as separate units. This affords flexibility in the number and order of chapters which may be used in a college algebra course. The tables at the back of the text include reciprocals, powers and roots of numbers, American Experience table of mortality, and four place logarithms. The text also contains answers to the odd-numbered problems. Anyone contemplating a new text in college algebra would do well to investigate the possibilities of this text.

T. E. RINE

ROGER BACON, IN LIFE AND LEGEND, by E. Westacott. Cloth. 12.7×19.0 cm. Pages xii+140. 1953. Philosophical Library, Inc., New York. N. Y. Price $3.75.

1954 would not characterize, even a genius, with "a philosopher, mathematician, physicist, linguist and theologian" though that is the description of this man of 1254+. At least his writings seem to warrant attention to such a variety of interests.

This volume, however, is not a biography in the usual sense. It might better be called a "copiously annotated handbook." Its two page bibliography will be a welcome help to the student who wants more than the sketchy suggestions found here.

Its major sections contain fourteen chapters, a Foreword, an Introduction, Thirteenth Century Background and an Epilogue. The chapters are: Life of Bacon; The Works of Bacon; Teachings; Oxford in the Time of Bacon; Philosopher and Scientist; Bacon in Tradition and Legend; Opus Majus; Why a Franciscan; Bacon and the Scriptures; The Master Mind; Bacon and Medicine; His Philosophy; In Memorium; and The Cipher of Roger Bacon. The chapters are very brief averaging but eight pages in length. There are five appendices, an eight paged double columned index. There is almost as much material in the ever-present footnotes as in the main body of the book.

The narration is tedious with over-attention to source material. There are many rather unrelated quotations that in the end give the reader the impression that the topic treated is left "loose ends." The book's most effective contribution may well come after the reader has briefed himself by reading some encyclopedia's paragraphs on Roger Bacon before using the *Roger Bacon, In Life and Legend.*

B. CLIFFORD HENDRICKS
Longview, Wash.

MORE POWER TO YOU, by Herman and Nina Schneider. Illustrated by Bill Ballantine. Cloth. 14.3×21.6 cm. Pages vi+128. William R. Scott, Inc. New York. Price $2.50.

The Schneiders are creditably known as writers on science in the language of the grade school child. In *More Power to You, from Windmills to Atomic Energy* they have presented the most difficult of their many undertakings. And as usual, at least, this reviewer thinks it's well done.

Their table of contents promises attention to: Power of: The Wind; Water;

Steam; Fire; Electricity and "New Engines and New Fuels." The method is by simple exposition, with over 100 line drawings, a number of working models and many experiments in which the child makes his own apparatus.

The child reader that has a native urge to know the "how of things" will find in this book's readable style attractive fare. The grown-ups may well get pointers from it on how to get and hold youthful attention. An eight page double columned index keys its offerings for the teacher who may use the book for supplementary readings.

B. CLIFFORD HENDRICKS

ALGEBRA 1 AND ALGEBRA 2, by C. A. Smith, *Chairman Mathematics, Central High School, St. Louis*, W. Fred Totten, *President Flint Junior College*, Harl R. Douglass, *Director of College of Education, University of Colorado*. Book I: Pages XII+500. 1954. Price $2.72. Book II: x+502, 1954. Price $2.72. Row, Peterson and Company, Evanston, Illinois.

The first book of this two book series is a ninth grade algebra text and the second is a text in intermediate algebra. Several features are present in both books. These include presenting algebra as a language, a Word Pick-up section at the start of each chapter, use of cartoons and other graphic devices to catch attention of pupils and to clarify processes, special emphasis on solving verbal problems with as much help as possible given by the text, variation and functional relationships emphasized, an abundance of practice and drill material with special exercises and topics designated for exceptional pupils, use of cross number puzzles for review purposes, a summary, chapter test, and cumulative review at the end of each chapter.

Book I covers the traditional topics through quadratics and includes a chapter on numerical trigonometry. It makes frequent use of red as a second color to attract attention to certain items, and the method appears to be effective. The authors state that they have used various devices to get the attention of the learner and to make the subject matter interesting to him. These characteristics are in evidence throughout the text and give it an appeal that should make the teacher want to take a second look.

Book II devotes the first six chapters to a review of the first course. It covers the topics usually found in this course including logarithms and the slide rule, quadratic functions and equations, systems of quadratics, series, and binomial theorem. It also includes numerical trigonometry, permutations, combinations, probability, and a chapter on rates of change. The coverage is adequate for a year course. For a semester course the teacher will need to select topics to fit his preference or the needs of his pupils. While this book does not make use of a second color in printing, it makes extensive use of illustrations and graphic devices and has an attractive appearance.

Both these books deserve careful examination by the teacher who is concerned with selecting a new text for first year algebra or for the second course in algebra.

GEORGE E. HAWKINS
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WATERPROOF COATING

Liquid rubber will waterproof anything from a boat bottom to a convertible car-top. Applied like paint, the rubber coating will provide a good grip on a baseball bat, will shockproof tools, weatherproof ignition wires, will even undercoat a car. The liquid, which is resistant to heat and cold, comes in black, green, and a transparent shade.

Reading is to the mind, what exercise is to the body.—ADDISSON, "*The Tatler.*"

A SCIENCE FAIR

LEONARD A. FORD
State Teachers College, Mankato, Minnesota

The purpose of a science fair is to stimulate teachers and students in their classroom work. If they know that the results of their work will be seen by others and that they are in competition with other teachers and students, better work is bound to be seen in the science classes. With this thought in mind the author has been instrumental in making a science fair an annual event in a region extending several hundred miles in area.

The purpose of this article is to convey to others some of the experiences gained from the fairs that have been conducted at Mankato State Teachers College. Certainly much values comes to the students and teachers when they can gather once a year and see what others are doing and enter into a day of excitement and scientific adventure.

A group of science teachers known as the Southern Minnesota Science Teachers League has regular meetings several times a year at the college. This group represents the people that give the greatest stimulus to the fair and they have their students working on projects and exhibits throughout the year. Although the group usually does not exceed twenty-five teachers they are the ones whose students bring the most projects to the fair.

Publicity for the fair goes out throughout the year with posters, mimeographed bulletins and personal letters. Newspapers in the area are given publicity material throughout the year as plans advance. A mailing list of all science teachers is obtained as early in the year as possible and these teachers are then kept informed of developments.

The fair has a feature presentation. A distinguished speaker who is a scientist shows slides or movies on some scientific subject. A panel of twelve judges selected from industry, education and medicine determines winners in various classifications. Judging is divided into general science 7th and 8th grade, general science 9th grade, chemistry, physics, biology, photography and mathematics. Prizes with a total value of $300, supplied by industrial concerns, were given to about 25 winners. Top prize had a value of $12.00. There were cash awards as well as handbooks. Students with the best exhibits were given blue ribbons and those with good exhibits were given red ribbons.

The 1954 fair had an attendance of 1500 high school students from 67 schools, 300 student exhibits were on display in addition to 20 industrial exhibits from such concerns as U. S. Steel Co., Minnesota Mining and Manufacturing Co., and many local companies that have scientific applications.

Throughout the year money was received from many industries that have an interest in activities of this type. Expenses were held to a minimum so that the money could be used for awards.

A feature of the fair that has proved very popular is the chemical magic show conducted by a group of college students. Mystery type experiments, explosions, color changes were performed along with a continual line of patter by the demonstrators. Students had worked on these experiments for many weeks, perfecting them to the point that they were almost foolproof. This feature was so popular that it was impossible to find room for the eager audience.

Although it is impossible to evaluate an event of this type on an objective basis all who participated in the fair this year are enthusiastic in its continuance. The fact that hundreds of students and teachers come from distances as great as 140 miles to take part in the events is an indication that the fair is carrying out its main objective: the stimulation of teachers and students in their work in science.

Foam-rubber seat is contoured to provide a more comfortable support for persons afflicted with ailments of pelvic and rectal regions. Having a scooped-out area that is said to help redistribute the body weight over a larger area, the seat may be used on any conventional chair or in automobiles.